## Modern Physics

## For second semester final year undergraduate students

Department of General Science
College of Basic Education
Salahaddin University-Erbil
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## Dear Student:

Modern Physics is a fundamental branch of the science of Physics. Modern Physics gives basic science to the students and researchers from all aspects, especially on microphysics, that is, science of the atom, nucleus and their composition. Modern Physics collects all the laws and ideas together concerned with the atom and its composition.
Modern Physics is a new branch of Physics started at the beginning of the $20^{\text {th }}$ century (from 1897 onwards) after the failer of classical Physics (before 1897) in interpreting the composition of matter.

## Dear reader;

This booklet outlines very short notes on modern Physics for final year undergraduate students of the Department of General Science, College of Basic Education, Salahaddin University-Erbil, Kurdistan Region - Iraq. It is only a guideline to more comprehensive knowledge of the modern physics. It is highly recommended that the student must read more from the textbooks mentioned in the references below, together with other sources in the internet.
I wish you a good luck and success.


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Moodle: Modern Physics
http://moodle.su.edu.krd/basiceducation/course/view.php?id=148

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References: سهرجاوهكان
1- Introduction to Modern Physics.Charles W.Fay. 2011.
2- Concept of Modern Physics. Sixths edition, Arthur Beiser,2003.
3- Introduction to modern physics, Volume I.R.B.Singh, 2009.
4- Modern Physics for Science \& Engineering.M.L.Burns.2012.
5- Modern Physics.Third edition, R.A.Serway, et.al. 2005.
6- Modern Physics.P.A.Tipler. 2008
7- http://ocw.tufts.edu/Course/36/Coursehome
8- http://galileo.phys.virginia.edu/classes/252/home.html


## Chapter One:

What is Physics?
Physics is the study of matter, energy and change. Physics has been divided into two branches:

Classical physics: Classical Physics is the name given to the physical theories generated before the twentieth century (before the year 1897). Classical physics is consisting of two areas, mechanics and electro-magnetism.
Modern physics: Modern Physics is the name given to the Theoretical advancements in physics since the beginning of the 20th Century (after the year 1897). Modern physics is known as Relativity and Quantum Mechanics.

## Classical Physics:

Science of physics began with Galileo. He was the first who based his ideas on experiments. After Galileo, Isaac Newton. Newton ideas dominated for about 200 years. Newton was the first to unify the theory of motion and gravitation. Newton linked the acceleration a body feels on the earth to the motion of celestial bodies. Newton proposed a simple set of principles;

1. An object will continue in a state of rest or uniform motion in a straight line unless acted on by an external force
2. The acceleration of an object is proportional to the net force applied to the object.
$\mathrm{F}=\mathrm{ma}$
The proportionality constant is a quantity defined as the mass, $m$.
3. For every force there is an equal and opposite force,

F12 = -F21
4. Newton's law of gravitation can be written as, $\mathrm{F}=\mathrm{Gm}_{1} \mathrm{~m}_{2} / \mathrm{r}^{2}$
Which simply states that two bodies of masses, $m_{1}$ and $m_{2}$ exert a force on each another proportional to the product of their masses and inversely proportional to the square of the distance between their centers. Newton also did work in optics and mathematics.

## Maxwell and Electro-Magnetism

The major work was done by James Clerk Maxwell, who brought together different experimental and theoretical of electricity and magnetism. Maxwell combined the work of Ampere, Gauss and Faraday into a set of equations that now called Maxwell's equations. Maxwell's equation's also showed that the electric and magnetic fields traveled in waves at the speed of light.

## Theories of Light:

## What is light?

Light is part of the electromagnetic spectrum, the spectrum is the collection of all waves, which include visible light, Microwaves, radio waves (AM, FM, SW ), X-Rays, and Gamma Rays.

Particle theory:_Newton, held the theory called particle theory that light was made up of tiny particles called corpuscles.

Electromagnetic theory: Maxwell showed that light are electromagnetic waves propagating in space with the speed of light. It states that light is made up of two components, electric and magnetic. These two components are perpendicular on each other and both perpendicular on the direction of the propagation.

Wave theory: In 1678, Huygens, stated that light was made up of waves vibrating up and down perpendicular to the direction of the light travels. This became known as 'Huygens' Principle'.

Quantum theory: In 1900 Max Planck proposed the existence of a light quantum, a finite packet of energy which depends on the frequency and velocity of the radiation.

From work of Plank on emission of light from hot bodies, in 1905 Einstein stated that light is composed of tiny particles called photons, and each photon has energy.

Light can exhibit both a wave theory and a particle theory at the same time. Much of the time, light behaves like a wave. Light waves are also called electromagnetic waves because they are made up of both electric $(E)$ and magnetic $(H)$ fields. Electromagnetic fields oscillate perpendicular to the direction of wave travel, and perpendicular to each other. Light waves are known as transverse waves as they oscillate in the direction traverse to the direction of wave travel.

## The Sine Wave



The sine wave is the fundamental waveform in nature. When dealing with light waves, we refer to the sine wave. The period ( $T$ ) of the waveform is one full 0 to 360 degree. The relationship of frequency and the period is given by the equation:
$f=1 / T$
$T=1 / f$

## The Electromagnetic Wave



Waves have two important characteristics - wavelength and frequency.
Wavelength: This is the distance between peaks of a wave. Wavelengths are measured in units of meters, When dealing with light, wavelengths are in the order of nanometres ( $1 \times 10^{-9}$ )
Frequency: This is the number of peaks that will travel past a point in one second. Frequency is measured in cycles per second. The term given to this is Hertz $(\mathrm{Hz})$ named after the 19th century discoverer of radio waves .
The speed of a wave can be found by multiplying wavelength and frequency together. The wave's speed is measured in units of length (distance) per second:
Speed $=$ Wavelength $\times$ Frequency
$\mathbf{C}=\mathbf{f} \mathbf{x} \boldsymbol{\lambda}$
If frequency is in Hz and wavelength in meters, then the speed will be in $\mathrm{m} / \mathrm{sec}$.

## The Speed of Light

The speed of light in a vacuum is a universal constant, about $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

## Photon Model of Light

As proposed by Einstein, light is composed of photons, a very small packets of energy. The reason that photons are able to travel at light speeds is due to the fact that they have no mass .A formula devised by Planck, is used to describe the relation between photon energy and frequency - Planck's Constant $(h)=6.63 \times 10^{-34}$ Joule-Second. and:

## $\boldsymbol{E}=\boldsymbol{h} \boldsymbol{x} \boldsymbol{f}$

$E$ is the photonic energy in Joules, $h$ is Planks constant and $f$ is the frequency in Hz Example: A photon emitted from the surface of the sun and traveled towards the earth. The average distance between the sun and the earth is $1.5 \times 10^{11} \mathrm{~m}$.Calculate the time of the travel of the photon?
Distance $=$ speed $x$ time or time $=$ distance/speed
Therefore time $=1.5 \times 10^{11} / 3 \times 10^{8}=500 \mathrm{sec}$.
Time $=500 / 60=8.3$ minutes .

## Questions:

1- Write on newton's laws of motion?
2- What is the difference between Classical physics and Modern Physics?
3- Two vehicles of masses 1000 and 2000 Kgs . The distance between their centers is 60 meters. Calculate the force of attraction between them? $\mathrm{G}=6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{Kg}^{-1}$ $\mathrm{s}^{-2}$

4- A proton in the nucleus of mass $1.67 \times 10^{-27} \mathrm{Kg}$ and an electron revolve around it of mass $9.1 \times 10^{-31} \mathrm{Kg}$. The distance between their centers is 0.5 Angstrom. Calculate the force of attraction between them? $1 \mathrm{~A}=10^{-10} \mathrm{~m}$
5- What is light? Explain.
6- State the theories of explanation of light.
7- A sine wave of frequency $\mathrm{f}=2 \times 10^{10} \mathrm{~Hz}$. Calculate the period T?
8- A photon of frequency $\mathrm{f}=12 \times 10^{14} \mathrm{~Hz}$ and a wavelength of $0.25 \times 10^{-6}$ meters. Calculate the speed of the photon.
9- Calculate the energy of a photon having the frequency of $12 \times 10^{14} \mathrm{~Hz}$ ? $\mathrm{h}=6.63 \times 10^{-}$ ${ }^{34}$ J.Sec.

10-Write on electromagnetic theory of light.

## Chapter Two: Relativity

1905 Einstein showed how measurements of time and space are affected by motion between an observer and what is being observed. Relativity connects space and time, matter and energy, electricity and magnetism. These links are important to our understanding of the physical universe.
Special Theory of Relativity: In 1905 Einstein put this theory and it is considered as the beginning of the modern physics.

## Postulates of Special Relativity

The two postulates of special relativity are:
1-The principle of relativity, states that the laws of physics are the same in all inertial frames of reference move with respect to one another.
2-The second postulate: The speed of light in free space has the same value in all inertial frames of reference. This speed is $3 \times 18^{8} \mathrm{~m} / \mathrm{s}$.
Time dilation: Special relativity indicates that, for an observer in an inertial frame of reference, a clock that is moving relative to him will be measured to tick slower than a clock that is at rest in his frame of reference.

In this reference frame, the time between the events is called proper time, and is labeled $\Delta \mathrm{t}_{0}$. In another reference frame, an observer will see the two events happen in different positions. The time between events is called observer time, and is labeled $\Delta \mathrm{t}$. The observer time is larger than the proper time. This effect is called time dilation. Both $\Delta t_{0}$ and $\Delta t$ are measured in seconds.

observer time $=\frac{\text { proper time }}{\sqrt{1-\left(\frac{\text { velocity }}{\text { speed of light }}\right)^{2}}}$
$\Delta t=\frac{\Delta t_{0}}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}}$
$\Delta t=$ the observer time, or two-position time (s) on earth
$\Delta \mathrm{t}_{0}=$ the proper time, or one-position time (s) on spaceship board
$\mathrm{v}=$ velocity ( $\mathrm{m} / \mathrm{s}$ )
$\mathrm{c}=$ speed of light $\left(3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)$

## Time Dilation Formula Questions:

1) Tanya boards a spaceship, and flies past Earth at 0.800 times the speed of light. Her twin sister, Tara, stays on Earth. At the instant Tanya's ship passes Earth, they both start timers. Tanya watches her timer, and after she sees 60.0 seconds have passed, she stops it. At that instant, how much time would Tara's timer say has passed?

## Answer:

$$
\begin{aligned}
& \Delta t=\frac{\Delta t_{0}}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}} \\
& \Delta t=\frac{60.0 \mathrm{~s}}{\sqrt{1-\left(\frac{0.800 c}{c}\right)^{2}}} \\
& \Delta t=\frac{60.0 \mathrm{~s}}{\sqrt{1-(0.800)^{2}}} \\
& \Delta t=\frac{60.0 \mathrm{~s}}{\sqrt{1-0.640}} \\
& \Delta t=\frac{60.0 \mathrm{~s}}{\sqrt{0.360}} \\
& \Delta t=\frac{60.0 \mathrm{~s}}{0.600} \\
& \Delta t=100 \mathrm{~s}
\end{aligned}
$$

2) Cosmic rays colliding with Earth's upper atmosphere produce high-energy particles called muons. An observer detects that a muon has been created, and observes that it reaches the surface of the Earth $20.0 \times 10^{-6}$ seconds later. The observer also determines
that the muon was moving at $2.97 \times 10^{8} \mathrm{~m} / \mathrm{s}$. In the muon's reference frame, how much time passed between its creation and reaching the surface of the Earth?
Answer:

$$
\begin{aligned}
\Delta t & =\frac{\Delta t_{0}}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}} \rightarrow \Delta t_{0}=\Delta t \sqrt{1-\left(\frac{v}{c}\right)^{2}} \\
\Delta t_{0} & =\Delta t \sqrt{1-\left(\frac{v}{c}\right)^{2}} \\
\Delta t_{0} & =\left(20.0 \times 10^{-6} s\right) \sqrt{1-\left(\frac{2.97 \times 10^{8} \mathrm{~m} / \mathrm{s}}{3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}}\right)^{2}} \\
\Delta t_{0} & =\left(20.0 \times 10^{-6} s\right) \sqrt{1-(0.990)^{2}} \\
\Delta t_{0} & =\left(20.0 \times 10^{-6} s\right) \sqrt{1-0.9801} \\
\Delta t_{0} & =\left(20.0 \times 10^{-6} s\right) \sqrt{0.0199} \\
\Delta t_{0} & \cong\left(20.0 \times 10^{-6} s\right)(0.141) \\
\Delta t_{0} & \cong 2.82 \times 10^{-6} \mathrm{~s}
\end{aligned}
$$

In the muon's reference frame, approximately $2.82 \times 10^{-6}$ seconds pass between when the muon is created and when it reaches the Earth's surface.

## Length Contraction Formula:

Special relativity states that the distance between two points can differ in different reference frames. The distance between points, and therefore the length, depends on the velocity of one reference frame relative to another. In one reference frame, an object being measured will be at rest. This is called the proper length, and is labeled $\Delta 1_{0}$. In another reference frame, an observer will see the object moving. The length of the object in this reference frame is observed length, and is labeled $\Delta l$. The observed length is always shorter than the proper length. This effect is called length contraction. Both $\Delta \mathrm{l}_{0}$ and $\Delta \mathrm{l}$ are measured in meters (m).
observed length $=($ proper length $) \sqrt{1-\left(\frac{\text { velocity }}{\text { speed of light }}\right)^{2}}$
$\Delta l=\Delta l_{0} \sqrt{1-\left(\frac{v}{c}\right)^{2}}$
$\Delta \mathrm{l}=$ the observed length, in the reference frame in which the object is moving (m)
$\Delta \mathrm{l}_{0}=$ the proper length, in the reference frame in which the object is at rest (m)
$\mathrm{v}=$ velocity $(\mathrm{m} / \mathrm{s})$
$\mathrm{c}=$ speed of light $\left(3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}\right)$

## Length Contraction Formula Questions:

1) A crew member of a spaceship measures the ship's length to be 100 m . The ship flies past Earth at a speed of 0.900 times the speed of light. If observers on Earth measure the length of the ship, what would they measure?
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Answer:
\(\Delta l=\Delta l_{0} \sqrt{1-\left(\frac{v}{c}\right)^{2}}\)
\(\Delta l=(100 m) \sqrt{1-\left(\frac{0.900 c}{c}\right)^{2}}\)
\(\Delta l=(100 \mathrm{~m}) \sqrt{1-(0.900)^{2}}\)
\(\Delta l=(100 \mathrm{~m}) \sqrt{1-(0.810)^{2}}\)
\(\Delta l=(100 \mathrm{~m}) \sqrt{1-0.810}\)
\(\Delta l=(100 \mathrm{~m}) \sqrt{0.190}\)
\(\Delta l \cong(100 \mathrm{~m})(0.436)\)
\(\Delta l \cong 43.6 \mathrm{~m}\)
```

The observers on Earth measure the length of the ship to be 43.6 m . This is less than the 100 m length measured in the reference frame of the ship's crew member.
2) Cosmic rays colliding with Earth's upper atmosphere produce high-energy particles called muons. An observer detects that a muon has been created 55.0 km above the surface of the Earth. Another observer detects the muon when it arrives at the surface. The observers determine that the muon was moving at $2.97 \times 10^{8} \mathrm{~m} / \mathrm{s}$. In the muon's reference frame, what was the distance between where it was created and the surface of the Earth?
Answer: The two positions to consider are the position at which the muon was created, and its arrival at the surface of the Earth. The distance between these positions in the reference frame of the observers is $\Delta l$. In the muon's reference frame, the distance between the points in the proper length, $\Delta l_{0}$. The distance in the reference frame of the observers is known, and so the distance in the muon's reference frame can be found by rearranging the length contraction formula:

$$
\begin{aligned}
& \Delta l=\Delta l_{0} \sqrt{1-\left(\frac{v}{c}\right)^{2}} \rightarrow \Delta l_{0}=\frac{\Delta l}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}} \\
& \Delta l_{0}=\frac{\Delta l}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}} \\
& \Delta l_{0}=\frac{55.0 \mathrm{~km}}{\sqrt{1-\left(\frac{2.97 \times 10^{8} \mathrm{~m} / \mathrm{s}}{3.0 \times 10^{8} \mathrm{~m} / \mathrm{s}}\right)^{2}}} \\
& \Delta l_{0}=\frac{55.0 \mathrm{~km}}{\sqrt{1-(0.990)^{2}}} \\
& \Delta l_{0}=\frac{55.0 \mathrm{~km}}{\sqrt{1-0.9801}} \\
& \Delta l_{0}=\frac{55.0 \mathrm{~km}}{\sqrt{0.0199}} \\
& \Delta l_{0} \cong \frac{55.0 \mathrm{~km}}{0.141} \\
& \Delta l_{0} \cong 390 \mathrm{~km} \\
& \Delta l_{0} \cong 390000 \mathrm{~m}
\end{aligned}
$$

In the muon's reference frame, the distance between where it is created and the surface of the Earth is approximately 390 km , or 390000 m . This is significantly longer than the contracted length, 55.0 km or 55000 m , measured by the observers.

## Special Relativity



## Twin Paradox:

Suppose one of two twins travels at near the speed of light to a distant star and returns to the earth. Relativity when he comes back, he is younger than his identical twin brother. . Hence, the brother who travels to the star is younger.

## General theory of relativity:

The two postulates of Einstein's general theory of relativity are:

- The laws of nature have the same form for observers in any frame of reference, whether accelerated or not.
- A gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects (This is the principle of equivalence.)
In 1916 Albert Einstein has established his famous theory known as the general theory of relativity
- The basis of this theory is the principle of equivalence, which states that:
- The principle of equivalence: there is no experiment of any kind that can distinguish uniformly accelerated motion from the presence of a gravitational field.
The general theory of relativity predicted many plhysicall phenomenom in the Universe
- First: The deviation of light in a gravitational field. The light beam when passes nearby large objects, deflects from its original passage. The discovery of double quasars is an optical illusion due to this effect, and known as gravitational lens effect.
- Second: The gravitational Red shift.
- Third: The precession of the perihelion of Mercury.
- Fourth: The time delay of electromagnetic waves that passes through a gravitational field ( Shapero time delay).
- Fifth: The existence of black holes.
- Sixth: Radiation of gravitational waves.

In principle, the general theory of relativity considers the gravity as the space-time curvature, and when you put a point matter on it, it vibrates and then ejects an elementary particle named Graviton.This elementary particle has never been detected.

## What is a Red Shift?

- A red shift is a shift in the frequency of a photon toward lower energy, or longer wavelength. The red shift is defined as the change in the wavelength of the light divided by the rest wavelength of the light, as follow
- $Z=$ (Observed wavelength - Rest wavelength)/(Rest wavelength)
- Note that positive values of $Z$ correspond to increased wavelengths (red shifts).
- Different types of red shifts have different causes.

- The stars ( the sun) emits radiation with continuous spectrum.
- As the light travels through the stars atmosphere, some of the light is absorbed by the same gas that emitted it but cooler.
- Therefore, the continuous emission spectrum has a few dark lines superimposed called Franhhouffer lines.
- It is these absorption lines that show the evidence of the red shift.
- Absorption lines in the optical spectrum of a supercluster of distant galaxies (below), as compared to absorption lines in the optical spectrum of the Sun (above). Arrows indicate redshift. Wavelength increases up towards the red and beyond (frequency decreases).



## Types of Red Shifts:

1- Cosmological red shift: This theory states that the quasars red shift is cosmological origin due to the expansion of the Universe. Using Hubble's law, the distance of the quasars can be determined.
The Cosmological Redshift is a redshift caused by the expansion of space. The wavelength of light increases as it traverses the expanding universe between its point of emission and its point of detection by the same amount that space has expanded during the crossing time.
2- Doppler relativistic red shift: This theory states that the red shift of quasars is Doppler origin, due to the motion of quasars, preceding from us.
The non existence of blue shift means all the quasars passed by us, long time ago.

- The Doppler Redshift results from the relative motion of the light emitting object and the observer. If the source of light is moving away from you then the wavelength of the light is stretched out, i.e., the light is shifted towards the red. These effects, individually called the blue shift, and the redshift are together known as doppler shifts.

Red Shift and Blue Shift:



## - Doppler \& Cosmological Red Shift

- Two different sources of redshift: Top, Doppler shift: the star moving to the left emits light that is blue shifted in the direction of the receiving antenna that the star approaches, and red shifted in the direction of the receiving antenna that the star is leaving. Center and bottom panels: cosmological expansion: The distance between the emitting star and both antennas increases while the light is propagating, increasing the wavelength of the light seen by both antennas.


3- Gravitational red shift: This theory states that the red shift of quasars is due to the general theory of relativity.
The Gravitational Redshift is a shift in the frequency of a photon to lower energy as it climbs out of a gravitational field.
It is not due to the motion of the quasars.Neither due to the expansion of the universe.This explains the non existence of blue shift.

## Causes of Red Shifts:

- The red shift is due to that the quasars are small objects of very large mass of very high dense gravity.
- Quasars will be converted into black holes and disappear.
- This explains the existence of white holes.


## Doppler Relativistic Red Shift:

- The formula can be stated as follow:
- $\lambda^{\prime} / \lambda=[(1+\mathrm{v} / \mathrm{c}) /(1-\mathrm{v} / \mathrm{c})]^{0.5}$, where
- $\lambda^{\prime}=$ wavelength measured on earth and
- $\lambda=$ wavelength emitted the star or Galaxy
- In term of the frequency:
- $\mathrm{f}^{\prime} / \mathrm{f}=[(1-\mathrm{v} / \mathrm{c}) /(1+\mathrm{v} / \mathrm{c})]^{0.5}$, and f is the frequency.


## Gravitational Redshift:

- Change of the wavelength of the photons, when they pass nearby the gravitational field.
- Thus, change in energy of the photon is equal to the potential energy. That is:
- $\mathrm{E}-\mathrm{E}^{\prime}=\mathrm{GMm} / \mathrm{R}$
- hc/ $\lambda-\mathrm{hc} / \lambda^{\prime}=\mathrm{GMm} / \mathrm{R}$
- $\mathrm{E}=\mathrm{mc}^{2}=\mathrm{hc} / \lambda$, thus $\mathrm{m}=\mathrm{h} / \lambda \mathrm{c}$
- Thus $\Delta \lambda / \lambda^{\prime}=G M / \mathrm{c}^{2} \mathrm{R}$.....Einstein Redshift
- The gravitation redshift is usually expressed in the units of $\mathrm{km} / \mathrm{sec}$, as follow
- $\quad \Delta \lambda / \lambda^{\prime}=\mathrm{v} / \mathrm{c}=\mathrm{GM} / \mathrm{c}^{2} \mathrm{R}$ from this:
- $\mathrm{V}=\mathrm{GM} / \mathrm{cR} \ldots .$.

Example:

- The gravitational redshift of a photon leaving the sun is:
- $\mathrm{V}=\mathrm{GM} / \mathrm{cR}=6.67 \times 10^{-8} \times 2 \times 10^{33} /\left(3 \times 10^{8} \times 7 \times 10^{10}\right)$

Thus $\mathrm{v}=0.6 \mathrm{~km} / \mathrm{sec}$

## Questions and answers:

Q1) What is relativity in Physics?
Relativity is a subject of Modern Physics put forword by Einstein in the period 1905 to 1916.
Q2) How many types of relativity theories in Physics?
Two types: 1-Special theory of relativity 1905 and 2- general Theory of relativity 1916.
Q3) what are the postulates of special theory of relativity?
The two postulates of special relativity are:
1-The principle of relativity, states that the laws of physics are the same in all inertial frames of reference move with respect to one another.
2-The second postulate: The speed of light in free space has the same value in all inertial frames of reference. This speed is $3 \times 18^{8} \mathrm{~m} / \mathrm{s}$.

Q4) What are the consequences of relativity theory:
The consequences ( results) of relativity theory are:
1- Time dilation; 2- length contraction; 3-mass dilation ; 4- Twin paradox
Q5) What are the principles of general theory of relativity?
The two postulates of Einstein's general theory of relativity are

- The laws of nature have the same form for observers in any frame of reference, whether accelerated or not.
- In the vicinity of any point, a gravitational field is equivalent to an accelerated frame of reference in the absence of gravitational effects (This is the principle of equivalence.)

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In 1916 Albert Einstein has established his famous theory known as the general theory of relativity

- The basis of this theory is the principle of equivalence, which states that:
- The principle of equivalence: there is no experiment of any kind that can distinguish uniformly accelerated motion from the presence of a gravitational field.

Q6)What are the predictions of the general theory of relativity?

## The general theory of relativity predicted many plhysical phenomenom in the Universe

- First: The deviation of light in a gravitational field. The light beam when passes nearby large objects, deflects from its original passage. The discovery of double quasars is an optical illusion due to this effect, and known as gravitational lens effect.
- Second: The gravitational Red shift.
- Third: The precession of the perihelion of Mercury.
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In principle, the general theory of relativity considers the gravity as the space-time curvature, and when you put a point matter on it, it vibrates and then ejects an elementary particle named Graviton.This elementary particle has never been detected.
Q7) what is of the redshifts? A red shift is a shift in the frequency of a photon toward lower energy, or longer wavelength. The red shift is defined as the change in the wavelength of the light divided by the rest wavelength of the light, as follow
$Z=$ (Observed wavelength - Rest wavelength)/(Rest wavelength)
Q8) write the types of redshift?
1- Cosmological redshift
2- Doppler relativistic redshift
3- Gravitational redshift
Q9) Derive the equation of gravitational redshift?
Change of the wavelength of the photons, when they pass nearby the gravitational field.
Thus, change in energy of the photon is equal to the potential energy. That is:
$\mathrm{E}-\mathrm{E}^{\prime}=\mathrm{GMm} / \mathrm{R}$
hc/ $\lambda-\mathrm{hc} / \lambda^{\prime}=\mathrm{GMm} / \mathrm{R}$
$\mathrm{E}=\mathrm{mc}^{2}=\mathrm{hc} / \lambda$, thus $\mathrm{m}=\mathrm{h} / \lambda \mathrm{c}$
Thus $\Delta \lambda / \lambda^{\prime}=\mathrm{GM} / \mathrm{c}^{2} \mathrm{R}$.....Einstein Redshift
The gravitation redshift is usually expressed in the units of $\mathrm{km} / \mathrm{sec}$, as follow
$\Delta \lambda / \lambda^{\prime}=\mathrm{v} / \mathrm{c}=\mathrm{GM} / \mathrm{c}^{2} \mathrm{R}$ from this we get:
$\mathrm{V}=\mathrm{GM} / \mathrm{cR}$
Q10) A star is moving away from Earth at a speed $0.825 c$. It emits radio waves with a wavelength of 0.525 m . What wavelength would we detect on Earth?

```
\(\lambda\) Observed \(=\lambda\) Source \([(1+\mathrm{v} / \mathrm{c}) /(1-\mathrm{v} / \mathrm{c})]^{0.5}\)
    \(=0.525[(1+0.825 \mathrm{c} / \mathrm{c}) /(1-0.825 \mathrm{c} / \mathrm{c})]^{0.5}\)
    \(=0.525 \times 3.23=1.7 \mathrm{~m}\) therefore the star is redshifted (increased wavelength)
```

Q11) Suppose a space probe moves away from Earth at a speed 0.350 c. It sends a radio-wave message back to Earth at a frequency of 1.50 GHz . At what frequency is the message received on Earth?

$$
\begin{aligned}
\mathrm{f} \text { Observed }= & \mathrm{f} \text { Source }[(1-\mathrm{v} / \mathrm{c}) /(1+\mathrm{v} / \mathrm{c})]^{0.5} \\
& =1.5 \mathrm{GHz}[1-0.350 \mathrm{c} / \mathrm{c}) /(1+\mathrm{o} .350 \mathrm{c} / \mathrm{c})]^{0.5} \\
& =1.5[0.65 / 1.35]^{0.5} \\
& =1.5 \times 0.69=1.04 \mathrm{GHz}
\end{aligned}
$$

Important: An observer of electromagnetic radiation sees relativistic Doppler effects if the source of the radiation is moving relative to the observer.
The wavelength of the radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer.
The shifted wavelength is described by the equation:
$\lambda$ Observed $=\lambda$ Source $[(1+\mathrm{v} / \mathrm{c}) /(1-\mathrm{v} / \mathrm{c})]^{0.5}$
Where $\lambda$ Observed is the observed wavelength, $\lambda$ Source is the source wavelength, and $v$ is the relative velocity of the source to the observer.

## Chapter Three: The Atom

Thomson in 1897 discovered the electrons and in 1899 Millikan calculated the charge of the electron to be $1.6 \times 10^{-19} \mathrm{C}$.

Rutherford was the first who discovered that the atoms of the matter consist of nucleus contain protons and neutrons and electrons revolving around the nucleus in orbits.

The most important study was conducted by Bohr and established a model of the atom of hydrogen. He stated that the atom of hydrogen consists of nucleus which contains one proton and one electron revolves around the nucleus in fixed orbits named energy states.

## Bohr postulates of the model of the atom of hydrogen:

The basic postulates of the Bohr theory as it applies to hydrogen atom are as follows:
i) The electron moves in stable circular orbits about the proton. The attractive force is the Coulomb force.
ii) Only certain orbits are stable. In a stable orbit, the electron does not radiate, so the energy is constant.
iii) Radiation is emitted when the electron makes a transition from one stable orbit to another, and $\Delta \mathrm{E}=h f$.
iv) The angular momentum of the electron in its orbit is quantized, $m v r=n \hbar$

The frequency $f$ of the photon emitted in the jump is independent of the frequency of the electron's orbital motion. The frequency of the light emitted is related to the change in the atom's energy and is given by the Planck-Einstein formula:
$\mathrm{E}_{\mathrm{i}}-\mathrm{E}_{\mathrm{f}}=\mathrm{hxf} \quad \mathrm{h}=$ Planck constant $=6.63 \times 10^{-34} \mathrm{~J} . \mathrm{S}$

where $E \mathrm{i}$ is the energy of the initial state and $E f$ is the energy of the final state, and $E \mathrm{i}$ $>E_{\mathrm{f}}$

The size of the allowed electron orbits is determined by an additional quantum condition imposed on the electron's orbital angular momentum. Namely, the allowed orbits are those for which the electron's orbital angular momentum about the nucleus is an integral multiple of $\hbar=\mathbf{h} / 2 \pi=1.0545718 \times 10^{-34} \mathrm{~m}^{\mathbf{2}} \mathbf{~ k g} / \mathrm{s}$
$\mathrm{m}_{\mathrm{e}} \mathrm{vr}=\mathrm{n} \hbar \quad \mathrm{m}_{\mathrm{e}} \mathrm{vr}=$ angular momentum

## Bohr Radius of the orbits in Hydrogen atom:

The equation of the radius of the orbits of the electron in hydrogen is
$\mathrm{r}_{\mathrm{n}}=\mathrm{n}^{2} \mathrm{a}_{\mathrm{o}}$
where n is the orbit number $\mathrm{n}=1,2,3,4, \ldots$ And $\mathrm{a}_{0}$ is the radius of the first orbit and $\mathrm{a}_{\mathrm{o}}=$ $5.3 \AA($ Angstrom $)=0.053 \mathrm{~nm}$.

## Energy of the orbits of Hydrogen atom:

The equation of calculating the energy levels (orbits) of hydrogen atom is
$E_{n}=-13.6 / n^{2} \mathrm{eV} \quad$ where $\mathrm{eV}=$ electron volt $=1.6 \times 10^{-19}$ Joule
It can be seen that the energy is negative, indicating bound electron-proton system
When $n=1$ then $E_{n}=-13.6 \mathrm{eV}$ and called ground state ( first excited state)
When $n=2$ then $E_{n}=-13.6 / 4=-3.4 \mathrm{eV}$ and so on.. .
Question: What is electron volt?


## Discuss the electronic and structural properties of an atom?

- An atom is composed of two regions: the nucleus, which is in the center of the atom and contains protons and neutrons, and the outer region of the atom, which holds its electrons in orbit around the nucleus.
- Protons and neutrons have approximately the same mass, about $1.67 \times 10-24$ grams, which scientists define as one atomic mass unit (amu) or one Dalton.
- Each electron has a negative charge (-1) equal to the positive charge of a proton (+1).
- Neutrons are uncharged particles found within the nucleus.
- atom: The smallest possible amount of matter which still retains its identity as a chemical element, consisting of a nucleus surrounded by electrons.
- proton: Positively charged subatomic particle forming part of the nucleus of an atom and determining the atomic number of an element. It weighs 1 amu .
- neutron: A subatomic particle forming part of the nucleus of an atom. It has no charge. It is equal in mass to a proton or it weighs 1 amu .
An atom is the smallest unit of matter that retains all of the chemical properties of an element. Atoms combine to form molecules, which then interact to form solids, gases, or liquids. For example, water is composed of hydrogen and oxygen atoms that have combined to form water molecules. Many biological processes are devoted to breaking down molecules into their component atoms so they can be reassembled into a more useful molecule.


## Atomic Particles

Atoms consist of three basic particles: protons, electrons, and neutrons. The nucleus (center) of the atom contains the protons (positively charged) and the neutrons (no charge). The outermost regions of the atom are called electron shells and contain the electrons (negatively charged). Atoms have different properties based on the arrangement and number of their basic particles.

The hydrogen atom $(\mathrm{H})$ contains only one proton, one electron, and no neutrons. This can be determined using the atomic number and the mass number of the element (see the concept on atomic numbers and mass numbers).


Structure of an atom: Elements, such as helium, depicted here, are made up of atoms. Atoms are made up of protons and neutrons located within the nucleus, with electrons in orbitals surrounding the nucleus.

Atomic Mass


Protons and neutrons have approximately the same mass, about $1.67 \times 10^{-24}$ grams. Scientists define this amount of mass as one atomic mass unit (amu) or one Dalton. Although similar in mass, protons are positively charged, while neutrons have no charge. Therefore, the number of neutrons in an atom contributes significantly to its mass, but not to its charge.
Electrons are much smaller in mass than protons, weighing only $9.11 \times 10^{-28}$ grams, or about $1 / 1800$ of an atomic mass unit. Therefore, they do not contribute much to an element's overall atomic mass. When considering atomic mass, it is customary to ignore the mass of any electrons and calculate the atom's mass based on the number of protons and neutrons alone.
Electrons contribute greatly to the atom's charge, as each electron has a negative charge equal to the positive charge of a proton. Scientists define these charges as " +1 " and " -1 ." In an uncharged, neutral atom, the number of electrons orbiting the nucleus is equal to the number of protons inside the nucleus. In these atoms, the positive and negative charges cancel each other out, leading to an atom with no net charge.

| Protons, Neutrons, and Electrons |  |  |  |
| :--- | :--- | :--- | :--- |
|  | Charge | Mass (amu) | Location |
| Proton | +1 | 1 | nucleus |
| Neutron | 0 | 1 | nucleus |
| Electron | -1 | 0 | orbitals |

Protons, neutrons, and electrons: Both protons and neutrons have a mass of 1 amu and are found in the nucleus. However, protons have a charge of +1 , and neutrons are uncharged. Electrons have a mass of approximately 0 amu , orbit the nucleus, and have a charge of -1 .

Questioin: Exploring Electron Properties: Compare the behavior of electrons to that of other charged particles to discover properties of electrons such as charge and mass.
Volume of Atoms
Accounting for the sizes of protons, neutrons, and electrons, most of the volume of an atomgreater than 99 percent-is, in fact, empty space. Despite all this empty space, solid objects do not just pass through one another. The electrons that surround all atoms are negatively charged and cause atoms to repel one another, preventing atoms from occupying the same space.
Question: Interactive: Build an Atom: Build an atom out of protons, neutrons, and electrons, and see how the element, charge, and mass change?
Atomic Number and Mass Number
The atomic number is the number of protons in an element, while the mass number is the number of protons plus the number of neutrons.
Question: Determine the relationship between the mass number of an atom, its atomic number, its atomic mass, and its number of subatomic particles

- Neutral atoms of each element contain an equal number of protons and electrons.
- The number of protons determines an element's atomic number and is used to distinguish one element from another.
- The number of neutrons is variable, resulting in isotopes, which are different forms of the same atom that vary only in the number of neutrons they possess.
- Together, the number of protons and the number of neutrons determine an element's mass number.
- Since an element's isotopes have slightly different mass numbers, the atomic mass is calculated by obtaining the mean of the mass numbers for its isotopes.
- mass number: The sum of the number of protons and the number of neutrons in an atom.
- atomic number: The number of protons in an atom.
- atomic mass: The average mass of an atom, taking into account all its naturally occurring isotopes.
- Atomic Number
- Neutral atoms of an element contain an equal number of protons and electrons. The number of protons determines an element's atomic number $(Z)$ and distinguishes one element from another. For example, carbon's atomic number $(Z)$ is 6 because it has 6 protons. The number of neutrons can vary to produce isotopes, which are atoms of the same element that have different numbers of neutrons. The number of electrons can also be different in atoms of the same element, thus producing ions (charged atoms). For instance, iron, Fe , can exist in its neutral state, or in the +2 and +3 ionic states.
- Mass Number
- An element's mass number (A) is the sum of the number of protons and the number of neutrons. The small contribution of mass from electrons is disregarded in calculating the mass number. This approximation of mass can be used to easily calculate how many neutrons an element has by simply subtracting the number of protons from the mass number. Protons and neutrons both weigh about one atomic mass unit or amu. Isotopes of the same element will have the same atomic number but different mass numbers.

- Atomic number, chemical symbol, and mass number: Carbon has an atomic number of six, and two stable isotopes with mass numbers of twelve and thirteen, respectively. Its average atomic mass is 12.11 .
- Scientists determine the atomic mass by calculating the mean of the mass numbers for its naturally-occurring isotopes. Often, the resulting number contains a decimal. For example, the atomic mass of chlorine ( Cl ) is 35.45 amu because chlorine is composed of several isotopes, some (the majority) with an atomic mass of 35 amu ( 17 protons and 18 neutrons) and some with an atomic mass of 37 amu ( 17 protons and 20 neutrons).

Given an atomic number ( Z ) and mass number (A), you can find the number of protons, neutrons, and electrons in a neutral atom. For example, a lithium atom ( $Z=3, A=7 \mathrm{amu}$ ) contains three protons (found from Z ), three electrons (as the number of protons is equal to the number of electrons in an atom), and four neutrons ( $7-3=4$ ).

## Questions \& Answers

Q1) Write Bohr postulates of Hydrogen Atom
i) The electron moves in stable circular orbits about the proton. The attractive force is the Coulomb force.
ii) Only certain orbits are stable. In a stable orbit, the electron does not radiate, so the energy is constant.
iii) Radiation is emitted when the electron makes a transition from one stable orbit to another, and $\Delta \mathrm{E}=h f$.
iv) The angular momentum of the electron in its orbit is quantized, $m v r=n \hbar$

Q2) If the radius of the first orbit of the electron in Hydrogen is $\mathrm{a}_{0}=0.53 \AA$. Calculate the radius of the third orbit?
We have $r_{n}=n^{2} a_{o} \quad$ then $r_{3}=3^{2} \times 0.53=9 \times 0.53=4.47 \AA$
Q3) Write the equation of the energy of the orbits of Hydrogen atom. Then calculate the energy of the first and fourth orbits in eV?
We have $E_{n}=-13.6 / n^{2}$ in eV
First orbit $n=1$, then $E_{1}=-13.6 / 1=-13.6 \mathrm{eV}$
Fourth orbit $n=4$, then $E_{4}=-13.6 / 4^{2}=-13.6 / 16=-0.85 \mathrm{eV}$
Q4) Calculate the energy difference between first and fourth orbits
$E_{1}=-13.6 \mathrm{eV}$ and $\mathrm{E}_{4}=-0.85 \mathrm{eV}$, then $\mathrm{E}_{4}-\mathrm{E}_{1}=-0.85-(-13.6)=-0.85+13.6=12.75 \mathrm{eV}$
Q5) Prove that the rest energy of an electron is equal to 0.511 MeV ?
$\mathrm{E}=\mathrm{m}_{\mathrm{e}} \mathrm{C}^{2}$
$\mathrm{E}=9.11 \times 10^{-31} \times\left(3 \times 10^{8}\right)^{2}$
$\mathrm{E}=8.2 \times 10^{-14} \mathrm{~J}$
$\mathrm{E}=8.2 \times 10^{-14} / 1.6 \times 10^{-19}$
$\mathrm{E}=0.511 \mathrm{eV}$
Q6) Calculate the rest energy of a proton?
$\mathrm{E}=\mathrm{m}_{\mathrm{p}} \mathrm{C}^{2}$
$\mathrm{E}=1.67 \times 10^{-27} \times\left(3 \times 10^{8}\right)^{2}$
$\mathrm{E}=1.5 \times 10^{-10} \mathrm{~J}$
$\mathrm{E}=1.5 \times 10^{-10} / 1.6 \times 10^{-19}$
$\mathrm{E}=938 \mathrm{MeV}$
Q7) What is the atom?
The smallest possible amount of matter which still retains its identity as a chemical element, consisting of a nucleus surrounded by electrons. An atom is composed of two regions: the nucleus, which is in the center of the atom and contains protons and neutrons, and the outer region of the atom, which holds its electrons in orbit around the nucleus.
Q8) Determine the relationship between the mass number of an atom, its atomic number, its atomic mass?
$\mathrm{A}=\mathrm{Z}+\mathrm{N}$ wher Z is number of protons in the nucleus and N is the number of neutrons in the nucleus.

## Chapter Four: Origin of Quantum Mechanics and Quantum Theory:

The history of quantum mechanics is a fundamental part of the history of modern physics. Quantum mechanics' began with a number of different scientific discoveries:

1- the 1838 discovery of cathode rays by Michael Faraday;
2- the 1859-60 black-body radiation problem by Kirchhoff;
3- the 1877 suggestion by Boltzmann that the energy states and radiation law
4- the discovery of the photoelectric effect by Heinrich Hertz in 1887
5- In 1900 quantum hypothesis by Max Planck that the energy E is proportional to the frequency $f$ as defined by the following formula:

$$
\mathrm{E}=\mathrm{hxf}
$$

where $h$ is called Planck's constant $=6.63 \times 10^{-34}$ J.S.

## Quantum theory of light:

Einstein in 1905, in order to explain the photoelectric effect previously reported by Hertz in 1887, postulated with Max Planck's quantum hypothesis that light itself is made of individual quantum particles, which in 1926 came to be called photons.
The photoelectric effect was observed upon shining light of particular wavelengths on certain materials, such as metals, which caused electrons to be ejected from those materials only if the light quantum energy was greater than the work function of the metal's surface.
The "quantum mechanics" was explained by the group of physicists including Max Born, Heisenberg, and Pauli, at the University of Göttingen in the early 1920s, and was first used by Born's 1924.
Boltzmann suggested in 1877 that the energy levels of a physical system, such as a molecule, could be discrete.
In 1900, the German physicist Max Planck reluctantly introduced the idea that energy is quantized in order to derive a formula for the observed frequency dependence of the energy emitted by a black body, called Planck's law.
These energy quanta are called "photons". The idea that each photon had to consist of energy in terms of quanta was a remarkable achievement; it effectively solved the problem of black-body radiation energy. In 1913, Bohr explained the spectral lines of the hydrogen atom, again by using quantization.
With decreasing temperature, the peak of the blackbody radiation curve shifts to longer wavelengths and also has lower intensities. The blackbody radiation curves (1862) at left are also compared with the early, classical limit model of Rayleigh and Jeans (1900) shown at right. The short wavelength side of the curves was already approximated in 1896 by the Wien distribution law ( $\lambda_{\max } \mathrm{T}=2898 \mu \mathrm{mK}$ ).

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Bohr's 1913 quantum model of the atom, which incorporated an explanation of Rydberg's 1888 formula,

Max Planck's 1900 quantum hypothesis, i.e. that atomic energy radiators have discrete energy values ( $\square=h \square$ ). Thomson's 1904 model, Einstein's 1905 light quanta postulate, and Rutherford's 1907 discovery of the atomic nucleus.

Note that the electron does not travel along the black line when emitting a photon. It jumps, disappearing from the outer orbit and appearing in the inner one and cannot exist in the space between orbits 2 and 3 .

In 1923, the French physicist de Broglie put forward his theory of matter waves by stating that particles can exhibit wave and particle characteristics. This theory was for a single particle and derived from special relativity theory. Quantum mechanics was born in 1925. When Erwin Schrödinger invented wave mechanics and the non-relativistic Schrödinger equation as an approximation to the generalized case of de Broglie's theory. Schrödinger subsequently showed that the two approaches were equivalent.
Heisenberg formulated his uncertainty principle in 1927.

## Photoelectric Effect:

## Photoelectric Effect



The photoelectric effect occurs when light above a certain frequency (the threshold frequency) is shone on metals like zinc, this causes electrons to escape from the zinc. The escaping electrons are called photoelectrons. It was shown in experiments that;

- the frequency of the light needed to reach a particular minimum value (depending on the metal) for photoelectrons to start escaping the metal
- the maximum kinetic energy of the photoelectrons depended on the frequency of the light not the intensity of the light
The above two observation can only be explained if the electromagnetic waves are emitted in packets of energy (quanta) called photons, the photoelectric effect can only be explained by the particle behavior of light. The photoelectric equation involves;
- $\mathrm{h}=$ the Plank constant $6.63 \times 10^{-34} \mathrm{~J} \mathrm{~s}$
- $f=$ the frequency of the incident light in hertz (Hz)
- $\Phi=$ the work function in joules $(\mathrm{J})=$ amount of the energy binding the electron to the metal.
- $\quad \mathrm{E}_{\mathrm{k}}=$ the maximum kinetic energy of the emitted electrons in joules (J)

$$
h f=\phi+E_{k}
$$

The energy of a photon of light $=\mathrm{hf}$ and the work function $(\phi)$ is the minimum energy required to remove an electron from the surface of the material.

So we can see from the equation above that if the light does not have a big enough frequency ( f ) so that the photon has enough energy to overcome the work function ( $\phi$ ), then no photoelectrons will be emitted. The above equation can be rearranged into the from

## $\mathbf{y}=\mathbf{m x}+\mathbf{c}$

So plotting a graph of frequency (f) on the x -axis and maximum kinetic energy $\left(\mathrm{E}_{\mathrm{k}}\right)$ on the y -axis will give a straight line graph. Where the slope is the Plank constant (h) and the $y$ intercept is the work function $(\phi)$, the intercept on the $x$ axis is the threshold frequency $f_{0}$.

$$
\begin{aligned}
& h f=\phi+E_{k} \\
& h f-\phi=E_{k} \\
& E_{k}=h f-\phi \\
& \downarrow \\
& y=m \dot{x}+c
\end{aligned}
$$



## Example 1:

Table I: Photoelectric Effect with 4 mm Aperture

|  | $\Delta$ Run \#1 | $\square$ Run \#1 |
| :---: | :---: | :---: |
|  | Frequency $\times 10^{\text {M }}$ <br> $(\mathrm{Hz})$ | Stopping Voltage <br> $(\mathrm{V})$ |
|  | 8.214 | 1.835 |
| 2 | 7.408 | 1.428 |
| 3 | 6.879 | 1.248 |
| 4 | 5.490 | 0.671 |
| 5 | 5.196 | 0.551 |

## EXAMPLE:

Actual data for a photoelectric effect experiment \{using sodium (Na)\} is given below. Use this data to derive a value for Planck's constant $h$ and the work function $\left(\mathrm{W}_{\mathrm{o}}\right)$ for Na .

| Light (Photon) Frequency $\left(10^{14} \mathrm{~Hz}\right)$ | Electron Kinetic Energy $\left(\mathbf{K E ~}_{\text {max }}\right)(\mathrm{eV})$ |
| :---: | :---: |
| 5.552 | 0.0085 |
| 5.996 | 0.182 |
| 6.517 | 0.408 |
| 7.138 | 0.695 |
| 7.889 | 0.972 |
| 8.817 | 1.309 |
| 9.993 | 1.848 |

## Ans:

Perform a linear least squares fit $\mathrm{y}=\langle\mathrm{a}\rangle \mathrm{x}+\langle\mathrm{b}\rangle$ since $\mathrm{KE}_{\max }=\mathrm{hf}-\mathrm{W}_{\mathrm{o}}$ is linear (see solid line in graph). The linear fit is a least squares fit from Excel (linear) trendline option.

$$
\begin{array}{ll}
<\mathrm{a}>=4.0973 \times 10^{-15} \mathrm{eV} \times 1.602 \times 10^{-19} \mathrm{~J} / \mathrm{eV}=\underline{\mathbf{6} .56 \times 10^{-34} \mathrm{~J} \mathrm{~s}} & {\left[h=6.63 \times 10^{-34} \mathrm{~J} \mathrm{~s}\right]} \\
<\mathrm{b}>=\underline{\mathbf{- 2 . 2 6 e V}} & {\left[\mathrm{W}_{\mathrm{o}}(\mathrm{Na})=-2.46 \mathrm{eV}\right]}
\end{array}
$$

Compton scattering, discovered by Compton, is the scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X-ray or gamma ray photon), called the Compton effect. Part of the energy of the photon is transferred to the recoiling electron. Inverse Compton scattering occurs when a charged particle transfers part of its energy to a photon.
Compton observed the scattering of x-rays from electrons in a carbon target and found scattered x-rays with a longer wavelength than those incident upon the target. The shift of the wavelength increased with scattering angle according to the Compton formula:

$$
\lambda_{f}-\lambda_{i}=\Delta \lambda=\frac{h}{m_{e} c}(1-\cos \theta)
$$

Compton explained and modeled the data by assuming a particle (photon) nature for light and applying conservation of energy and conservation of momentum to the collision between the photon and the electron. The scattered photon has lower energy and therefore a longer wavelength according to the Planck relationship. Here the quantity $\mathrm{h} / \mathrm{m}_{\mathrm{e}} \mathrm{C}$ is known as Compton wavelength and equal to $\boldsymbol{\lambda}_{\mathbf{c}}=\mathbf{2 . 4 3 \times 1 0} \mathbf{- 1 2}^{\mathbf{- 1 2}} \mathbf{m}$
At a time (early 1920's) when the particle (photon) nature of light suggested by the photoelectric effect was still being debated, the Compton experiment gave clear and independent evidence of particle-like behavior. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".

## X- Rays:

X-rays were discovered in 1895 by Roentgen.He found that a beam of high-speed electrons striking a metal target produced a new and extremely penetrating type of radiation. Within months of Roentgen's discovery the first medical x-ray pictures were taken, and within several years it became evident that x -rays were electromagnetic vibrations similar to light but with extremely short wavelengths and great penetrating power. Estimates obtained from the diffraction of x rays by a narrow slit showed x-ray wavelengths to be about $10^{-10} \mathrm{~m}$, which is of the same order of magnitude as the atomic spacing in crystals. William Henry Bragg and William Lawrence Bragg in England suggested using single crystals such as calcite as natural three-dimensional gratings, the periodic atomic arrangement in the crystals constituting the grating rulings. A particularly simple method of analyzing the scattering of x-rays from parallel crystal planes was proposed by W. L. Bragg in 1912.
Conditions of producing X-Rays:

1. The three things needed to create x-rays are a source of electrons, a means of accelerating the electrons to high speeds, and a target for the accelerated electron to interact with.
2. X-rays are produced when the free electrons cause energy to be released as they interact with the atomic particles in the target.

X-rays are produced by bombarding a metal target (copper, tungsten) with energetic electrons having energies of 50 to 100 keV . The minimum continuous x-rays wavelength, L min, is found to depend only on the tube voltage, $V$. All of the incident electron's kinetic energy is converted to electromagnetic energy in the form of a single x-rays photon. For this case we have:
$\mathrm{eV}=\mathbf{h f}=\mathbf{h c} / \lambda \min$
Then $\lambda \min =h c / e V$
Then $\lambda \min =6.63 \times 10^{-34} \times 3 \times 10^{8} / \mathrm{V} \times 1.6 \times 10^{-19}$
$\lambda \min =12.4 \times 10^{-7} / V=1.24 / \mathrm{MV} \quad \mathrm{M}=\mathrm{Mega}=1 \times 10^{6}$

## Example: The Compton Shift for Carbon

X-rays of wavelength $\lambda=0.200 \mathrm{~nm}$ are aimed at a block of carbon. The scattered x -rays are observed at an angle of 45.0 to the incident beam. Calculate the increased wavelength of the scattered x-rays at this angle.
Solution:
The shift in wavelength of the scattered x-rays is given by $\lambda_{\mathrm{f}}-\lambda_{\mathrm{i}}=2.43 \times 10^{-12}(1-\cos (\theta)$
$\lambda_{\mathrm{f}}-\lambda_{\mathrm{i}}=2.43 \times 10^{-12}(1-\cos (45))=7.11 \times 10^{-13} \mathrm{~m}=0.000711 \mathrm{~nm}$.
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## Characteristic X-Rays



Characteristic x-rays are emitted from heavy elements when their electrons make transitions between the lower atomic energy levels. The characteristic x-ray emission which is shown as two sharp peaks in the illustration at left occur when vacancies are produced in the $\mathrm{n}=1$ or K -shell of the atom and electrons drop down from above to fill the gap. The x-rays produced by transitions from the $\mathrm{n}=2$ to $\mathrm{n}=1$ levels are called K-alpha x-rays, and those for the $\mathrm{n}=3 \rightarrow 1$ transition are called K-beta xrays.

Transitions to the $\mathrm{n}=2$ or L -shell are designated as $L \mathrm{x}$-rays ( $\mathrm{n}=3 \rightarrow 2$ is L alpha, $\mathrm{n}=4 \rightarrow 2$ is L-beta, etc. ). The continuous distribution of x -rays which forms the base for the two sharp peaks at left is called "bremsstrahlung" radiation.

X-ray production typically involves bombarding a metal target in an $\underline{x}$-ray tube with high speed electrons which have been accelerated by tens to hundreds of kilovolts of potential. The bombarding electrons can eject electrons from the inner shells of the atoms of the metal target. Those vacancies will be quickly filled by electrons dropping down from higher levels, emitting x-rays with sharply defined frequencies associated with the difference between the atomic energy levels of the target atoms.

The frequencies of the characteristic x-rays can be predicted from the Bohr model. Moseley measured the frequencies of the characteristic x-rays from a large fraction of the elements of the periodic table and produced a plot of them which is now called a "Moseley plot".

Characteristic x-rays are used for the investigation of crystal structure by x-ray diffraction. Crystal lattice dimensions may be determined with the use of Bragg's law in a Bragg spectrometer.

$\Delta$ Figure 2-1. Simple x-ray tube.

## Bragg's Law ( X-Rays diffraction):

X-Rays are scattered when it falls on a plane of crysral atoms. The equation of the scattering is called Bragg's law. As shown.


## de Broglie particle- wave duality:

By the early 1920s scientists recognized that the Bohr theory contained many inadequacies:

- It failed to predict the observed intensities of spectral lines.
- It had only limited success in predicting emission and absorption wavelengths for multi-electron atoms.
- It failed to provide an equation of motion governing the time development of atomic systems starting from some initial state.
- It overemphasized the particle nature of matter and could not explain the newly discovered wave-particle duality of light.
- It did not supply a general scheme for "quantizing" other systems, especially those without periodic motion.
The first bold step toward a new mechanics of atomic systems was taken by Louis Victor de Broglie in 1923 . He postulated that because photons have wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties.

According to de Broglie, electrons had a dual particle-wave nature. Accompanying every electron was a wave. He concluded that the wavelength and frequency of a matter wave associated with any moving object were given by
$\lambda=\mathrm{h} / \mathrm{p}$
Were $\mathrm{p}=\mathrm{mv}$ is the momentum of the particle, and

## E=hf

where $h$ is Planck's constant, $p$ is the momentum, and $E$ is the total energy of the particle.
Example : Determine the wavelength of an electron of mass $9.11 \times 10^{-31} \mathrm{Kg}$, having a velocity of $5.65 \times 10^{7} \mathrm{~m} / \mathrm{s}$ ? and calculate the energy?

## Wave Nature of the Electron

- 1925 - Louis de Broglie

(Nobel prize in physics in 1929)
- Not only electromagnetic waves can be sometimes considered as particles (photons)
- Very small particles (electrons) might also behave as waves under the proper circumstances
$\lambda=\frac{h}{m v} \longleftarrow \begin{aligned} & \text { mass and velocity } \\ & \text { of the particle }\end{aligned}$
12


## Heisenberg uncertainty principle:

In 1927 Heisenberg developed his idea concerned with the wave-particle duality that:
It is impossible to determine simultaneously the position and momentum of particle.
If a measurement of a position is made with $d x$ and simultaneously a measurement of the momentum is made with dp in the x - direction, the product can never be smaller than $\hbar / 2$. Thus
$\Delta \mathrm{x} . \Delta \mathrm{p} \geq \hbar / 2 \quad$ position - momentum uncertainty principle
This means as one uncertainty increase, the other one decrease. And as one uncertainty approaches infinity, the other one must approached zero.
The uncertainty in energy and time is also in the form:
$\Delta \mathrm{E} . \Delta \mathrm{t} \geq / \hbar / 2$ energy-time uncertainty principle

## Questions Chapter Four

Q1) what are the discoveries of quantum mechanics?
Q2) Describe the quantum theory of light?

Q3) What is Photoelectric effect?
Q3) Draw the diagram of the photoelectric effect?
Q4) write the equation of the photoelectric effect?
Q5) What is Compton scattering?
Q6) Draw the diagram of Compton scattering?
Q7) Write the equation of Compton scattering?
Q8) What is X-rays?
Q9) Write the conditions of producing X-Rays?
Q10) Derive the equation of calculation the minimum wavelength of $X$-rays production?
Q11) Draw the graph of the characteristics of X-rays and explain?
Q12) what is Bragg's law? Write the equation and draw the diagram?
Q13) Write the equation of de Broglie wavelength and explain?

## Chapter 5: One electron model

Rydberg equation:
When an electron jumps from an outer orbit to an inner-orbit it will radiate or emit a photon. The wavelength of the emitted radiation can be calculated by an equation called Rydberg equation as follow
$1 / \lambda=R\left(1 / n_{i}^{2}-1 / n_{f}^{2}\right)$
................. Rydberg Equation
$\mathrm{R}=1.1 \times 10^{7} \mathrm{~m}^{-1} \quad$ is known as Rydberg constant.

## Line spectrum of hydrogen atom:

In 1914, Bohr proposed a theory of the hydrogen atom which explained the origin of its spectrum and which also led to an entirely new concept of atomic structure. According to this theory, the wavelengths of the hydrogen spectrum could be calculated by Rydberg formula shown above.
When an atom absorbs a quantum of energy, it is said to be in an excited state relative to its normal (ground) state. When an excited atom returns to the ground state, it emits light. The various colors(lines) of the emitted light of definite wavelengths, is called a line spectrum.
The spectrum of hydrogen Emission or absorption processes in hydrogen give rise to series, which are sequences of lines corresponding to atomic transitions, each ending or beginning with the same atomic state in hydrogen. The various series of lines are named according to the lowest energy level involved in the transitions that give rise to the lines.
The Lyman series involve jumps to or from the ground state ( $\mathrm{n}=1$ );
The Balmer series (in which all the lines are in the visible region) corresponds to $\mathrm{n}=2$,
The Paschen series to $n=3$,
The Brackett series to $n=4$, and
The Pfund series to $\mathrm{n}=5$.



By setting $n_{\mathrm{f}}$ to 1 and letting $n_{\mathrm{i}}$ run from 2 to infinity, the spectral lines known as the Lyman series converging to 91 nm are obtained. In the same manner, the other series of spectral lines may be obtained using the values of $n_{\mathrm{f}}$ and $n_{\mathrm{i}}$ in the following table:

| $\boldsymbol{n}_{\mathbf{f}}$ | $\boldsymbol{n}_{\mathbf{i}}$ | Name | Wavelength |
| :--- | :--- | :--- | :--- |
| 1 | $2 \rightarrow \infty$ | $\underline{\text { Lyman series }}$ | 91 nm |
| 2 | $3 \rightarrow \infty$ | $\underline{\text { Balmer series }}$ | 365 nm |
| 3 | $4 \rightarrow \infty$ | $\underline{\text { Paschen series }}$ | 821 nm |
| 4 | $5 \rightarrow \infty$ | $\underline{\text { Brackett series }}$ | 1459 nm |
| 5 | $6 \rightarrow \infty$ | $\underline{\text { Pfund series }}$ | 2280 nm |
| 6 | $7 \rightarrow \infty$ | $\underline{\text { Humphreys series }}$ | 3283 nm |

The Lyman series is in the ultraviolet while the Balmer series is in the visible and the Paschen, Brackett, Pfund, and Humphreys series are in the infrared.

Example1: The electron in a hydrogen atom at rest makes a transition from $\mathrm{n}=2$ orbit to the $\mathrm{n}=1$ ground state.
Fined the wavelength, frequency and energy in eV?
Solution: Using Rydberg equation for $\mathrm{ni}=2$ and $\mathrm{nf}=1$
$1 / \lambda=1.1 \times 10^{7} \times(1 / 1-1 / 4)=3.3 \times 10^{7} / 4$
Thus $\lambda=1.215 \times 10^{-7} \mathrm{~m}=121.5 \mathrm{~nm}$
The frequency is $\mathrm{f}=\mathrm{c} / \lambda=3 \times 10^{8} / 1.215 \times 10^{-7}=2.47 \times 10^{15} \mathrm{~Hz}$
The energy $\mathrm{E}=\mathrm{hf}=6.63 \times 10^{-34} \times 2.47 \times 10^{15}=16.38 \mathrm{X}^{-19}$ Joule
$1 \mathrm{eV}=1.6 \times 10^{-19}$ Joule, thus $\mathrm{E}=16.38 \times 10^{-19} / 1.6 \times 10^{-19}=10.2 \mathrm{eV}$.
Example 2: The Balmer series of the Hydrogen atom is the electron transition to the orbit $\mathrm{n}=2$.
Find the longest wavelength of the photon emitted and calculated its energy?

پ.ی.د. محمد عزیز سعيد - بهشى زانستى كَشتى - كوّليّزَى پـهروهردهى بنهرهِتى - زانكوّى سهلاحهددين هلوليّر

The longest wavelength means maximum wavelength $\lambda \max$. And transition from $\mathrm{n}=3$.
Thus $1 / \lambda \max =\mathrm{R}((1 / 4-1 / 9)=5 R / 36$
Therefore $\lambda \max =36 / 5 \mathrm{R}=36 /(5 \mathrm{x} 1.1 \mathrm{x} 107)=656.3 \mathrm{~nm}$
This is in the red region of the visible spectrum.
Energy of the photon Ephoton $=\mathrm{hc} / \lambda$ max
Ephoton $=6.63 \times 10^{-34} \times 3 \times 10^{8} / 656.3 \times 10^{-9}=3.03 \times 10^{-19}=1.89 \mathrm{eV}$

## Chapter 6: Schrodinger Equation

The Schrödinger equation is a linear partial differential equation that describes the wave function or state function of a quantum-mechanical system. It is a key result in quantum mechanics, and its discovery was a significant landmark in the development of the subject. The equation is named after Erwin Schrödinger, who derived the equation in 1925, and published it in 1926.
Time dependent Schrodinger equation is

$$
i \hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t)=\left[\frac{-\hbar^{2}}{2 m} \nabla^{2}+V(\mathbf{r}, t)\right] \Psi(\mathbf{r}, t)
$$

Time independent Schrodinger equation is :

$$
\left[\frac{-\hbar^{2}}{2 m} \nabla^{2}+V(\mathbf{r})\right] \Psi(\mathbf{r})=E \Psi(\mathbf{r})
$$

Wavefunction: Is a mathematical function and it is the solution of Schrodinger equation.

## Particle in a 1-Dimensional box

A particle in a 1-dimensional box is a fundamental quantum mechanical approximation describing the translational motion of a single particle confined inside an infinitely deep well
The particle in a box is a common application of a quantum mechanical model to a simplified system consisting of a particle moving horizontally within an infinitely deep well. The solutions to the problem give possible values of E and $\psi$ that the particle can possess. E represents allowed energy values and $\psi(\mathrm{x})$ is a wave function, which when squared gives us the probability of locating the particle at a certain position within the box at a given energy level.
To solve the problem for a particle in a 1-dimensional box, we must:

[^0]2. Solve the Schrödinger Equation
3. Define the wave function
4. Define the allowed energies

## Step 1: Define the Potential Energy V



A particle in a $1 D$ infinite potential well of dimension LL .
The potential energy is 0 inside the box $(\mathrm{V}=0$ for $0<\mathrm{x}<\mathrm{L})$ and goes to infinity at the walls of the box $(\mathrm{V}=\infty$ for $\mathrm{x}<0$ or $\mathrm{x}>\mathrm{L})$. We assume the walls have infinite potential energy to ensure that the particle has zero probability of being at the walls or outside the box. Doing so significantly simplifies our later mathematical calculations as we employ these boundary conditions when solving the Schrödinger Equation.

## Step 2: Solve the Schrödinger Equation

The time-independent Schrödinger equation for a particle of mass $m$ moving in one direction with energy $E$ is
$\left(-\hbar^{2} / 2 \mathrm{~m}\right) \mathrm{d}^{2} \psi(\mathrm{x}) / \mathrm{d}^{2} \mathrm{x}+\mathrm{V}(\mathrm{x}) \psi(\mathrm{x})=\mathrm{E} \psi(\mathrm{x})$
with

- $\hbar$ is the reduced Planck Constant where $\hbar=\mathrm{h} / 2 \pi$
- $m$ is the mass of the particle
- $\psi(x)$ is the stationary time-independent wavefunction
- $\mathrm{V}(\mathrm{x})$ is the potential energy as a function of position
- $E$ is the energy, a real number

This equation can be modified for a particle of mass $m$ free to move parallel to the x -axis with zero potential energy ( $\mathrm{V}=0$ everywhere) resulting in the quantum mechanical description of free motion in one dimension:
$\left(-\hbar^{2} / 2 m\right) d^{2} \psi(x) / d^{2} x=E \psi(x)$


This equation has been well studied and gives a general solution of:

$$
\psi(\mathrm{x})=\mathrm{A} \sin (\mathrm{kx})+\mathrm{B} \cos (\mathrm{kx})
$$

where $\mathrm{A}, \mathrm{B}$, and k are constants.

## Step 3: Define the wave function

The solution to the Schrödinger equation we found above is the general solution for a 1dimensional system. We now need to apply our boundary conditions to find the solution to our particular system.
According to our boundary conditions, the probability of finding the particle at $\mathrm{x}=0$ or $\mathrm{x}=\mathrm{L}$ is zero.
When $x=0$ them $\sin (0)=0$, and $\cos (0)=1$;
therefore, $B$ must equal 0 to fulfill this boundary condition giving:
$\psi(\mathrm{x})=\mathrm{A} \sin (\mathrm{kx})$
We can now solve for our constants (A and $k$ ) systematically to define the wavefunction.

## Solving for $\mathbf{k}$

Differentiate the wavefunction with respect to x :
$\mathrm{d} \psi / \mathrm{dx}=\mathrm{kA} \cos (\mathrm{kx})$
$d^{2} \psi / d x^{2}=-k^{2} A \sin (k x)$
Since $\psi(x)=A \sin (k x)$, then
$\mathrm{d}^{2} \psi \mathrm{dx}=-\mathrm{k}^{2} \psi$
If we then solve for $k$ by comparing with the Schrödinger equation above, we find:
$\mathrm{k}=\left(8 \pi^{2} \mathrm{mE} / \mathrm{h}^{2}\right)^{\wedge}{ }^{\wedge .5}$
Now we plug k into our wave function:
$\psi=\mathrm{A} \sin \left(\left(8 \pi^{2} \mathrm{mE} / \mathrm{h}^{2}\right)^{\wedge 0.5} \mathrm{x}\right)$

## Solving for $\mathbf{A}$

To determine A, we have to apply the boundary conditions again. Recall that the probability of finding a particle at $x=0$ or $x=L$ is zero.
When $\mathrm{x}=\mathrm{L}$ :
$0=\mathrm{A} \sin \left(\left(8 \pi^{2} \mathrm{mE} / \mathrm{h}^{2}\right)^{\wedge 0.5} \mathrm{~L}\right)$
This is only true when
$\left(8 \pi^{2} \mathrm{mE} / \mathrm{h}^{2}\right)^{\wedge 0.5} \mathrm{~L}=\mathrm{n} \pi$
where $\mathrm{n}=1,2,3 \ldots$
Plugging this back in gives us:

$\psi=\mathrm{A} \sin (\mathrm{n} \pi \mathrm{x} / \mathrm{L})$
To determine A , recall that the total probability of finding the particle inside the box is 1 , meaning there is no probability of it being outside the box. When we find the probability and set it equal to 1 , we are normalizing the wave function.
$\int \psi^{2} d x=1$
For our system, the normalization looks like:
$A^{2} \int \sin ^{2}(n \pi x / L) d x=1$
Using the solution for this integral from an integral table, we find our normalization constant, A:
$A=\sqrt{ }(2 / L)$
Which results in the normalized wavefunction for a particle in a 1-dimensional box:
$\psi=\sqrt{ }(2 / L) \sin (n \pi x / L)$

## Step 4: Determine the Allowed Energies

Solving for E results in the allowed energies for a particle in a box:
$\mathrm{En}=\mathrm{n}^{2} \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}$
This is an important result that tells us:

1. The energy of a particle is quantized and
2. The lowest possible energy of a particle is NOT zero.
3. This is called the zero-point energy and means the particle can never be at rest because it always has some kinetic energy.
This is also consistent with the Heisenberg Uncertainty Principle: if the particle had zero energy, we would know where it was in both space and time.

## What does all this mean?

The wavefunction for a particle in a box at $\mathrm{n}=1$ and $\mathrm{n}=2$ energy levels look like this:



The probability of finding a particle a certain spot in the box is determined by squaring $\psi$. The probability distribution for a particle in a box at the $\mathrm{n}=1$ and $\mathrm{n}=2$ energy levels looks like this:


Notice that the number of nodes (places where the particle has zero probability of being located) increases with increasing energy .
Also note that as the energy of the particle becomes greater, the quantum mechanical model breaks down as the energy levels get closer together and overlap, forming a continuum. This continuum means the particle is free and can have any energy value.
At such high energies, the classical mechanical model is applied as the particle behaves more like a continuous wave.

Therefore, the particle in a box problem is an example of Wave-Particle Duality.

Example: What is the $\Delta \mathrm{E}$ between the $\mathrm{n}=4$ and $\mathrm{n}=5$ states for an F 2 molecule trapped within in a one-dimension well of length 3.0 cm ?

## Solution

Since this is a one-dimensional particle in a box problem, the particle has only kinetic energy $(\mathrm{V}=0)$, so the permitted energies are:
$E n=n^{2} h^{2} / 8 \mathrm{~mL}^{2}$
with $\mathrm{n}=1,2, \ldots \mathrm{n}=1,2, \ldots$
The energy difference between $n=4$ and $n=5$ is then $\Delta \mathrm{E}=\mathrm{E} 5-\mathrm{E} 4=\left(5^{2} \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}\right)-\left(4^{2} \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}\right)$

Using the mass of F2 ( $37.93 \mathrm{amu}=6.3 \times 10^{-26} \mathrm{~kg}$ ) and the length of the box ( $\mathrm{L}=3 \times 3.0 \times 10^{-2} \mathrm{~m} 2$ ):
$\Delta \mathrm{E}=9 \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}=9\left(6.63 \times 10^{-34} \mathrm{~kg} \cdot \mathrm{~m}^{2} \cdot \mathrm{~s}^{-1}\right)^{2} / 8\left(6.3 \times 10^{-26} \mathrm{~kg}\right)\left(3.0 \times 10^{-2} \mathrm{~m}^{2}\right)^{2}$
$\Delta \mathrm{E}=8.70 \times 10^{-39} \mathrm{~J}=8.7 \times 10^{-39} / 1.6 \times 10^{-19}=5.44 \times 10^{-20} \mathrm{eV}$

## Important Facts to Learn from the Particle in the Box

- The energy of a particle is quantized. This means it can only take on discreet energy values.
- The lowest possible energy for a particle is $\underline{\text { NOT zero (even at } 0 \mathrm{~K} \text { ). This means }}$ the particle always has some kinetic energy.
- The square of the wave function is related to the probability of finding the particle in a specific position for a given energy level.
- The probability changes with increasing energy of the particle and depends on the position in the box.
The particle in a box model describes a particle free to move in a small space surrounded by impenetrable barriers.
The particle in the box model system is the simplest application of the Schrödinger equation.


Figure : The barriers outside a one-dimensional box have infinitely large potential, while the interior of the box has a constant, zero potential.


Figure : A plot of $\psi \mathrm{n}(\mathrm{x})$ for the first four wave functions.

This figure shows part of the energy-level diagram for the particle in a box. The occurrence of discrete or quantized energy levels is characteristic of a bound system, that is, one confined to a finite region in space.

## Zero Point Energy

An interesting point is that $\mathrm{E}_{1}>0$, whereas the corresponding classical system would have a minimum energy of zero.
This is a recurrent phenomenon in quantum mechanics. The residual energy of the ground state, that is, the energy in excess of the classical minimum, is known as zero point energy.
The kinetic energy, hence the momentum, of a bound particle cannot be reduced to zero. The minimum value of momentum is found by $\mathrm{E}_{1}=\mathrm{P}^{2} / 2 \mathrm{~m}$, giving $\mathrm{P}_{\text {min }}= \pm \hbar / 2 \mathrm{~L}$.
This can be expressed as an uncertainty in momentum given by $\Delta \mathrm{p} \approx \hbar / \mathrm{L}$.
Coupling this with the uncertainty in position, $\Delta \mathrm{x} \approx \mathrm{L}$, from the size of the box, we can write
$\Delta \mathrm{x} \Delta \mathrm{p} \approx \hbar$
This is in accord with the Heisenberg uncertainty principle.
The particle in-abox eigen functions are given by $B=0$ and $k=n \pi / L$ $\psi_{\mathrm{n}}(\mathrm{x})=\mathrm{A} \sin (\mathrm{n} \pi \mathrm{x} / \mathrm{L})$
with $\mathrm{n}=1,2,3 \ldots$
These, like the energies, can be labeled by the quantum number $n$.
The constant A , can be adjusted so that $\psi \mathrm{n}(\mathrm{x})$ is normalized. The normalization condition is, in this case,

$$
\int\left[\psi_{\mathrm{n}}(\mathrm{x})\right]^{2} \mathrm{dx}=1
$$

## Chapter 8: Nuclear Structure

The nucleus of all atoms are composed of two types of particles
1- Protons, positive charged particles
2- Neutrons, zero charged particles
In describing the atomic nucleus, we must know:

- The atomic number $(\mathrm{Z})$ which is equal to the number of protons in the nucleus.
- The neutron number $(\mathrm{N})$ which is equal to the number of neutrons in the nucleus.
- The mass number (A) which is equal to the number of protons and neutrons in the nucleus. Thus
$\mathbf{A}=\mathbf{Z}+\mathbf{N}$
To present a nucleus on any element, we usually use a symbol, such as


The nucleus of all atoms contains the same number of protons. But sometimes it contains different number of neutrons and this is called Isotopes. Thus;
The isotopes of an element have the same $\mathbf{Z}$ value but different $\mathbf{N}$ and $\mathbf{A}$ values. For example


Isotopes of Carbon


Charge of proton $=$ Charge of electron $=1.6 \times 10^{-19} \mathrm{C}$
Mass of proton $=1.67 \times 10^{-27} \mathrm{Kg}$
Charge of neutron $=0$
Mass of neutron $=1.6 .7 \times 10^{-27} \mathrm{Kg}$
Charge of electron $=1.6 \times 10^{-19} \mathrm{C}$
Mass of electron $=9.1 \times 10^{-31} \mathrm{Kg}$
Atomic mass unit: it is the mass of Carbon 12 and is known as (u)
So mas of Carbon 12 is $=1 \mathrm{u}$
Thus Mass of proton $=1.007 \mathrm{u}$
Mass of neutron $=1.008 \mathrm{u}$
Sometimes, atomic mass unit is expressed in atomic rest energy as follow
$\mathrm{E}=\mathrm{mc}^{2}$
For proton $E=1.67 \times 10^{-27} \times(3 \times 108)^{2}=1.5 \mathrm{x}^{-10}$ Joule
1 joule $=1.6 \times 10^{-19}$ electron volt $(\mathrm{eV})$
Then $\mathrm{E}=1.5 / 1.6 \times 109=931 \mathrm{MeV}$
Then $1 \mathrm{u}=931 \mathrm{MeV} / \mathrm{c}^{2}$
Nucleus size ( radius)
Rutherford has found that the radius of the nucleus can be defined as
$r=r_{0} \times A^{1 / 3}$ where $r_{0}=1 \times 10^{-15} \mathrm{~m}=1$ fermi ( fm ) and $\mathrm{A}=$ mass number.
Therefore the volume of the nucleus is in spherical shape;
$\mathrm{V}=4 / 3 \pi \mathrm{r}^{3}=4 / 3 \pi \mathrm{r}_{0}{ }^{3} \mathrm{~A}$

## Binding Energy:

The binding energy of any nucleus can be calculated from the equation $\mathrm{E}_{\mathrm{b}}(\mathrm{MeV})=\left[\mathrm{ZM}_{\mathrm{H}}+\mathrm{Nm}_{\mathrm{n}}-\mathrm{M}_{\mathrm{A}}\right] \mathrm{x} 931 \mathrm{MeV}$
$\mathrm{M}_{\mathrm{H}}=$ atomic mass of Hydrogen
$\mathrm{M}_{\mathrm{A}}=$ atomic mass of the element and $\mathrm{m}_{\mathrm{n}}=$ mass of the neutron.

## Nuclear Stability:

Most elements have isotopes. For stable isotopes, A graph of number of neutrons is versus the number of protons shows the stable isotopes, this graph is often called the Nuclear Belt of Stability. The plot indicates that lighter nuclides (isotopes) are most stable when the neutron/proton ratio is $1 / 1$. This is the case with any nucleus that has up to 20 protons. As the atomic number increases beyond 20, a different trend becomes apparent. Stable isotopes have a higher neutron to proton ratio, rising to $1.5 / 1$ for elements having atomic numbers between 20 and 83 .

## Nuclear Force:

There are four types of forces between nucleons in the nucleus;
1- Gravitational Force between the masses of the nucleons
2- Electric force between protons (Columb's Law) Since the E. Force is > than the gravitational
3- Weak nuclear force between protons and neutrons
4- Strong nuclear force strongly attractive, very short range distance ( $10^{-15} \mathrm{~m}$ ). this force is necessary to keep the nucleus stable.
Note: Electric force > Gravitational force; nuclear force > gravitational force and electric force.


## Radioactivity:

Radioactivity is the process of spontaneous emission of radiation by the nucleus of some elements. It is the process of decay or disintegration of unstable nuclei.
There are three types of radiation emitted by radioactive substances:
1- Alpha particles $(\alpha)$, in which the emitted particles are the nucleus of Helium.
2- Beta particles $(\beta)$,in which the emitted particles either electrons or positrons.
3- Gamma rays $(\gamma)$,in which the emitted are rays of photons.
The equation of radioactive decay ( N ) or the number of radioactive nuclei present at time ( t ) is:
$\mathrm{N}=\mathrm{N}_{\mathrm{o}} \mathrm{e}^{-\lambda \mathrm{t}}$
And No $=$ number of radioactive nuclei at time $=0$ and $\lambda=$ decay constant $=$ the probability per unit time that the nucleus will decay .

## Half-life:

The half-life of a radioactive substance is the time it takes half of a given number of radioactive nuclei to decay. Then;
$\mathrm{N}=\mathrm{No} / 2$ and $\mathrm{t}=\mathrm{T}_{1 / 2}$
$\mathrm{N}_{\mathrm{o}} / 2=\mathrm{N}_{\mathrm{o}} \mathrm{e}^{-\lambda \mathrm{T} 1 / 2}$

Therefore $\mathrm{T}_{1 / 2}=\operatorname{Ln}(2) / \lambda ; \mathrm{T}_{1 / 2}=0.693 / \lambda$
The decay rate ( $R$ ) is
$\mathrm{R}=\mathrm{dN} / \mathrm{dt}=\mathrm{N}_{\mathrm{o}} \lambda \mathrm{e}^{-\lambda \mathrm{t}}$
And $N_{0} \lambda=$ Ro $=$ decay rate at time $t=0$, then
$R=R_{0} e^{-\lambda t}$
The radioactive activity is usually measure in unit of Curie and
$1 \mathrm{Ci}=3.7 \times 10^{10}$ decay/ sec. some times millicurie and microcurie are also used.

## Exercises

1) How fast a meter stick be moving if its length is observed to shrink
to 0.5 m ? Ans ( 0.87 C )
$\mathrm{L}=\mathrm{Lo}\left[1-\mathrm{V}^{2} / \mathrm{C}^{2}\right]^{\wedge} 0.5$
$0.5=1 \mathrm{x}\left[1-\mathrm{V}^{2} / \mathrm{C}^{2}\right]^{\wedge} 0.5$
Therefore; $\mathrm{V}=0.87 \mathrm{C}$
2) Show that the rest energy of an electron is equal to 0.511 MeV ?
$E=m_{e} C^{2}$
$\mathrm{E}=9.11 \times 10^{-31} \times\left(3 \times 10^{8}\right)^{2}$
$\mathrm{E}=8.2 \times 10^{-14} \mathrm{~J}$
$\mathrm{E}=8.2 \times 10^{-14} / 1.6 \times 10^{-19}$
$\mathrm{E}=0.511 \mathrm{MeV}$
3) Calculate the rest energy of a proton? Ans ( 938 MeV )
$\mathrm{E}=\mathrm{m}_{\mathrm{p}} \mathrm{C}^{2}$
$\mathrm{E}=1.67 \times 10^{-27} \times\left(3 \times 10^{8}\right)^{2}$
$\mathrm{E}=1.5 \times 10^{-10} \mathrm{~J}$
$\mathrm{E}=1.5 \times 10^{-10} / 1.6 \times 10^{-19}$
$\mathrm{E}=938 \mathrm{MeV}$
4) The peak sensitivity of human eye for Sun light is 500 nm ( bluegreen light of the Sun. Calculate the surface temperature of the Sun? Ans ( $\mathrm{T}=5800 \mathrm{~K}$ )
Tx $\lambda \max =2898 \mu \mathrm{~m} . \mathrm{K} \quad$ Wein displacement law
$\mathrm{T}=2898 \times 10^{-6} / \lambda \max =2898 \times 10^{-6} / 500 \times 10^{-9}$
$\mathrm{T}=5800 \mathrm{~K}$
5) Photoelectrons released from Zinc by ultraviolet light were stopped by a voltage of 4.3 volts. Find the maximum kinetic energy Kmax and maximum velocity Vmax for these electrons? Ans ( Kmax= $6.9 \times 10^{-19} \mathrm{~J} ; \mathrm{Vmax}=1.2 \times 10^{6} \mathrm{~m} / \mathrm{sec}$ )
$K \max =\mathrm{eV}=1.6 \times 10^{-19} \times 4.3=6.9 \times 10^{-19} \mathrm{~J}$
$\mathrm{eV}=\mathrm{m}_{\mathrm{e}} \mathrm{V}^{2}$ max $/ 2$
$\mathrm{Vmax}=\left[2 \mathrm{eV} / \mathrm{m}_{\mathrm{e}}\right]^{\wedge} 0.5$
Vmax $=\left[2 \times 1.6 \times 10^{-19} \times 4.3 / 9.11 \times 10^{-31}\right]^{\wedge} 0.5$
$V \max =1.2 \times 10^{6} \mathrm{~m} / \mathrm{sec}$
6) Find the shortest wavelength of the Balmer series of the emitted photon and calculate its largest (maximum) energy in eV ? Ans (: $\lambda \min =364.6 \mathrm{~nm}$ (Ultraviolet); $\operatorname{Emax}=3.4 \mathrm{eV}$ )
$1 / \lambda \min =\operatorname{Rx}\left(1 / n_{f}^{2}-1 / n^{2}{ }_{i}\right)$
$\mathrm{R}=$ Rydberg constant $=1.1 \times 10^{7} \mathrm{~m}^{-1}$
For Balmer series $\mathrm{n}_{\mathrm{f}}=2$ and for shortest wavelength $\mathrm{n}_{\mathrm{i}}=\infty$
Then; $1 / \lambda \min =\operatorname{Rx}(1 / 4)$
$\lambda \min =4 / \mathrm{R}=364.6 \mathrm{~nm}$
$\operatorname{Emax}=\mathrm{hc} / \lambda$ min
Emax $=6.63 \times 10^{-34} \times 3 \times 10^{8} / 364.6 \times 10^{-9}$
Emax $=0.05455 \times 10^{-17} \mathrm{~J}$
Emax $=0.05455 \times 10^{-17} / 1.6 \times 10^{-19}$
Emax $=3.4 \mathrm{eV}$
7) A free electron has a wave function $\Psi(x)=\sin \left(5 \times 10^{10} \mathrm{x}\right)$. Determine
a- The electron's de-broglie wavelength
b- Momentum
c- Kinetic energy
d- Speed of the electron
The wave function in x - direction is in the form

$$
\Psi(\mathrm{x})=\mathrm{A} \sin (\mathrm{kx})
$$

Therefore $\mathrm{A}=1 \mathrm{~m}$ and $\mathrm{k}=5 \times 10^{10} \mathrm{~m}^{-1}$
a) $\mathrm{K}=2 \pi / \lambda$

$$
\begin{aligned}
& \lambda=2 \pi / \mathrm{k}=2 \times 3.14 / 5 \times 10^{10}=1.26 \times 10^{-10} \mathrm{~m}=1.26 \AA \\
& 1 \AA=1 \times 10^{-10} \mathrm{~m}
\end{aligned}
$$

b) Momentum $\mathrm{P}=\hbar \mathrm{k}=\mathrm{hk} / 2 \pi$
$P=6.63 \times 10^{-34} \times 5 \times 10^{10} / 2 \times 3.14$
$\mathrm{P}=5.26 \times 10^{-24} \mathrm{Kg} . \mathrm{m} / \mathrm{sec}$
c) $\mathrm{E}=\hbar^{2} \mathrm{k} / 2 \mathrm{~m}=\mathrm{h}^{2} \mathrm{k} / 4 \pi^{2} \mathrm{x} 2 \mathrm{~m}$
$\mathrm{E}=\left(6.63 \times 10^{-34}\right)^{2} \times 5 \times 10^{10} /\left(4 \times 3.14^{2} \times 2 \times 9.1 \times 10^{-31}\right)$
E $1.53 \times 10^{-17} \mathrm{~J}=1.53 \times 10^{-17} / 1.6 \times 10^{-19}=95.2 \mathrm{eV}$
8) Prove that the amplitude (A) of the wavefunction of a particle in one dimensional box is $(2 / \mathrm{L})^{\wedge}{ }^{0.5}$ ?
The wave function in x -direction is $\Psi(\mathrm{x})=\mathrm{A} \sin (\mathrm{kx})$
And $\mathrm{k}=\mathrm{n} \pi / \mathrm{L}$
Then $\Psi(x)=A \sin (n \pi x / L) \quad 0<x<L ; n=1,2,3, \ldots .$.
To determine the value of (A) we must normalize the probability, that is, all probabilities sum to one.

$$
1=\int|\Psi(\mathrm{x})|^{2} \mathrm{dx}=\mathrm{A}^{2} \int \sin ^{2}(\mathrm{n} \pi \mathrm{x} / \mathrm{L}) \mathrm{dx}
$$

But $\sin ^{2}(\mathrm{n} \pi x / L)=(1-\cos (2 n \pi x / L)) / 2$
$1=\mathrm{A}^{2} \int(1-\cos (2 \mathrm{n} \pi \mathrm{x} / \mathrm{L})) / 2 \mathrm{dx}$
$1=A^{2} / 2 \int d x-A^{2} / 2 \int \cos (2 n \pi x / L) d x$
The last term is zero; then
$1=A^{2} L / 2$ then $A^{2}=2 / L$
Therefore $A=(2 / L)^{\wedge 0.5}$
9) Calculate the photon energy of wavelength 400 nm emitted by photoelectric effect?

$$
\begin{aligned}
\mathrm{E}= & \mathrm{hf}=\mathrm{hc} / \lambda=6.63 \times 10^{-34} \times 3 \times 10^{8} / 400 \times 10^{-9} \\
& =4.97 \times 10^{-19} \mathrm{~J} \\
& =4.97 \times 10^{-19} / 1.6 \times 10^{-19}=3.1 \mathrm{eV}
\end{aligned}
$$

10) The decay constant of a radioactive nuclei is $1.4 \times 10^{-11} \mathrm{~S}^{-1}$. Calculate the half life?
$\mathrm{T}_{1 / 2}=0.693 / \lambda$
$\mathrm{T}_{1 / 2}=0.693 / 1.4 \times 10^{-11}$
Then $\mathrm{T}_{1 / 2}=5 \times 10^{10} \mathrm{sec}$.
Then $\mathrm{T}_{1 / 2}=5 \times 10^{10} / 3.16 \times 10^{7}=1600$ years
Because 1 year $=365 \times 24 \times 60 \times 60=3.16 \times 10^{7} \mathrm{sec}$.
11) Calculate the binding energy of deuteron, which consist of a proton and a neutron?
The atomic mass unit of proton $\mathrm{M}(\mathrm{H})=1.007825 \mathrm{u}$
The atomic mass unit of neutron $\mathrm{m}_{\mathrm{n}}=1.008665 \mathrm{u}$
The atomic mass unit of deuteron $\mathrm{M}_{2}=2.014102 \mathrm{u}$
Using the equation of binding for deuteron:
$\mathrm{E}_{\mathrm{b}}=\left[\mathrm{M}(\mathrm{H})+\mathrm{m}_{\mathrm{n}}-\mathrm{M}_{2}\right] \times 931.494 \mathrm{MeV} / \mathrm{u}$
$\mathrm{E}_{\mathrm{b}}=[1.007825 \mathrm{u}+1.008665 \mathrm{u}-2.014102 \mathrm{u}] \mathrm{x} 931$
$\mathrm{E}_{\mathrm{b}}=0.002388 \mathrm{x} 931.494=2.224 \mathrm{MeV}$
This means that in order to separate the proton and neutron of the Deuteron, we must add 2.224 MeV energy to it. If the binding Energy of the nucleus is zero, then the proton and neutrons will separate spontaneously without adding energy. This nucleus will be radioactive.
12) A small object of mass 1 mg is moving between two walls separated by 1 cm . calculate the minimum energy level?
The Bohr energy level is
$\mathrm{En}=\mathrm{n}^{2} \pi^{2} \hbar^{2} / 2 \mathrm{~mL}^{2}=\mathrm{n}^{2} \mathrm{~h}^{2} / 8 \mathrm{~mL}^{2}$
The minimum energy level is $\mathrm{n}=1$; then

$$
\begin{aligned}
& \mathrm{E}_{1}=(6.63 \times 10-34)^{2} /\left[8 \times 1 \times 10^{-6} \times\left(1 \times 10^{-3}\right)^{2}\right] \\
& \mathrm{E}_{1}=5.49 \times 10^{-58} \mathrm{~J}=5.49 \times 10^{-58} / 1.6 \times 10^{-19}=3.4 \times 10^{-39} \mathrm{eV} .
\end{aligned}
$$

13) The threshold wavelength for potassium is 564 nm .
a- What is the work function of Potassium?
Work function $\Phi=\mathrm{hf}_{\mathrm{t}}=\mathrm{hc} / \lambda_{\mathrm{t}}=1240 \mathrm{eV} \mathrm{nm} / 564 \mathrm{~nm}$
Then $\Phi=2.2 \mathrm{eV}$.
b- What is the stopping potential when light of $1=400 \mathrm{~nm}$ is incident on Potassium?
$\mathrm{eVo}=\mathrm{hf}-\Phi$
$\Phi=2.2 \mathrm{eV}$ and $\mathrm{hf}=\mathrm{hc} / \lambda=\left(6.63 \times 10^{-34} \times 3 \times 10^{8} / 400 \times 10^{-}\right.$ ${ }^{9} \times 1.6 \times 10^{-19}$ ) $=3.1 \mathrm{eV}$
$\mathrm{eVo}=3.1 \mathrm{eV}-2.2 \mathrm{eV}$
Then $\mathrm{Vo}=0.9$ Volt is the stopping potential.

بابهتى موّديّرن فيزيكس بابهتيّكى سهرهكيه له زانستى فيزيا. ئهم بابهته بنهماى زانستى تـوواو دهبهخشيّت به قوتابى و تويّرَهران له هلموو بابهتدكانى زانست بهتايبه تى له بوارى مايكرِّفّفيزيكس واتا
 و يِيّكهاتهكانى و شيّوازى كاركردنى كوّكردوّتهوهو به رֶوونى ده يخاته بـهرده م خويّنهران. موّديّرن فيزيكس ( فيزياى نوىّ يان فيزياى سهردهم ) زانستيّيكى نويّيه له سهردتاى سالّى 1897) ددستى پِيّكرد دواى ئهوهى زانستى فيزياى كلاسيك ( كوّن) پیيّش سالّى (1897) نـهيتوانى تفسيرى پِيّكهاتهى ماده بكات.
قوتابى بهريِّز :

ئـهم ناميلكهيهى بهردهستت تهنها كورتهيهكه له سهر بابهتى فيزياى موّديّرن بوّ قوّناغى چوارهمى بهشى


 بخويّنيّتهوه و زياتر زانست وهربگريّت. به هيواى سهركهوتن ماموّستا: پ.ى.د. محمد عزيز سعيد زانستى گشتى- پـهروهر ردهى بنهرِهتى

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## فيزياى نوك"



بهشى زانستى كشتى - كوّليّزى بدروهر رهى بنهرهـتى
زانكوّى سهلاحهدددين - ههوليّر

وانهبيّز : ب.ى.د. محمل عزيز سحيد

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\text { سالّى ئككاليمى } 2022 \text { - } 2023
$$

## كانونى دووهم 2023


[^0]:    1. Define the Potential Energy, $V$
