# Classical Mechanics

## For first year undergraduate students


**Department of General Science**

**College of Basic Education**

**Salahaddin University-Erbil**

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**Academic Year: 2022/2023**

**September 2022**

**Dear Student/Reader**

This booklet outlines very short notes on classical mechanics for first year undergraduate students of the Department of General Science, College of Basic Education, Salahaddin University-Erbil, Kurdistan Region - Iraq. It is only a guideline to more comprehensive knowledge of the classical physics. It is highly recommended that the student must read more from the textbooks mentioned in the course book, together with other sources in the internet.
I wish you a good luck and success.

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**September 2022**

بەشی یەکەم : ئاراستەکان و نا ئاراستەکان

http://www.jfinternational.com/ph/vectors-scalars.html

**VECTORS AND SCALARS**

In physics and all science branches quantities are categorized in two ways. ***Scalars*** and ***vectors*.** They are used for to define quantities.

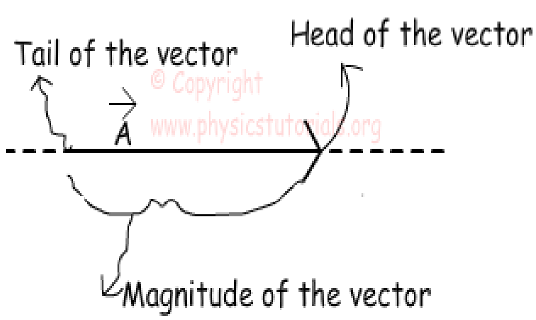
We can use scalars in just indication of the magnitude; they are only numerical value of that quantity.

A **vector** is an **oriented quantity**; it has **magnitude** and **direction** like velocity, force and displacement.
**Scalars** have **no direction** associated to them, **only magnitude**, like time, temperature, mass and energy.
**Vectors** are represented by **arrows** where the **length of the arrow** is drawn proportionally to the **magnitude** of the **vector**.
The letters denoting **vectors** are written in boldface.

However, if we talk about the vectors we should consider more than numeric value of the quantities. Vectors are explained in detail below.

**VECTORS**

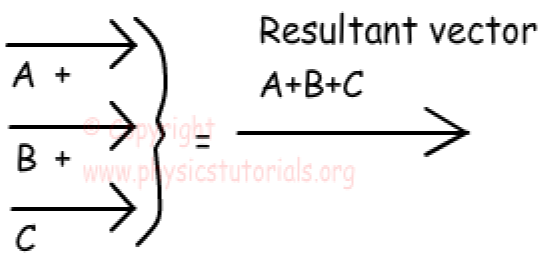
Vectors are used for some quantities having both magnitude and direction. We will first learn the properties of vectors and then pass to the vector quantities. You will be more familiar with concepts after learning vectors. Look at the given shape which is a vector having magnitude and direction.



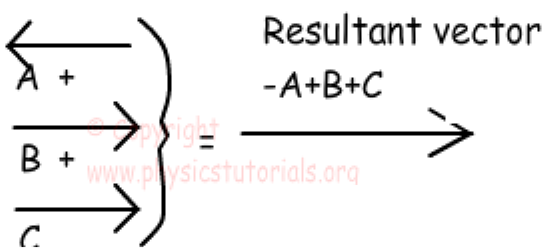

Head of the vector shows the direction and tail shows the starting point. We can change the position of the vector however, we should be careful not to change the direction and magnitude of it. In next subject we will learn how to add and subtract vectors. Moreover, we will learn how to find the X and Y components of a given vector using a little bit trigonometries.

**ADDITION OF VECTORS**

Look at the picture given below. It shows the classical addition of three vectors. We can add them just like they are scalars. However, you should be careful, they are not scalar quantities. They have both magnitude and direction. In this example their magnitudes and directions are the same thus; we just add them and write the resultant vector.



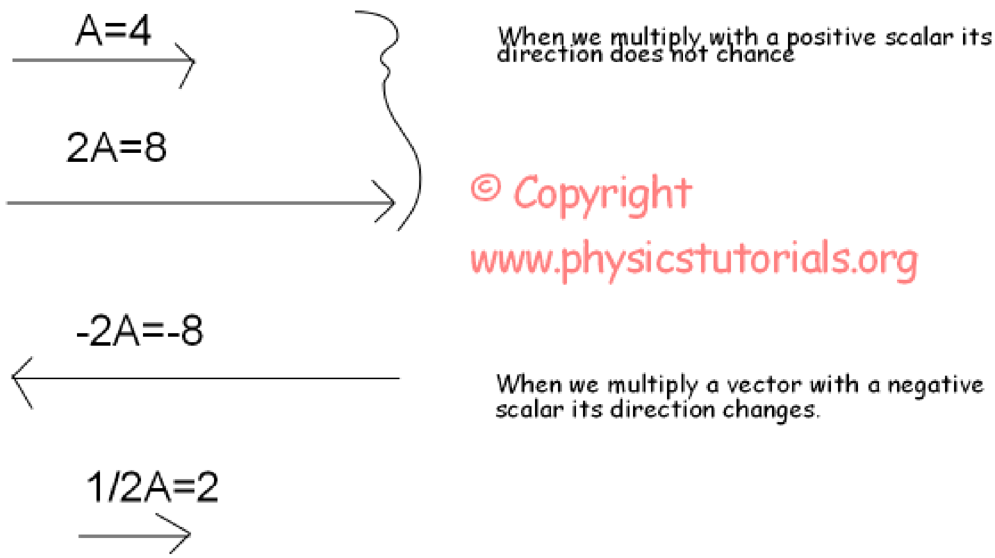

Let's look at a different example. In this example as you can see the vector A has negative direction with respect to vectors B and C. So, while we add them we should consider their directions and we put a minus sign before the vector A. As a result our resultant vector becomes smaller in magnitude than the first example.




**MULTIPLYING A VECTOR WITH A SCALAR**

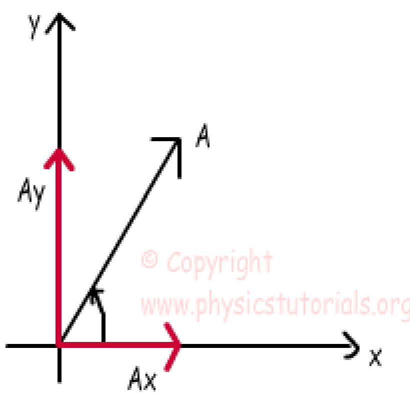
When we multiply a vector with a scalar quantity, if the scalar is positive than we just multiply the scalar with the magnitude of the vector. But, if the scalar is negative then we must change the direction of the vector. Example given below shows the details of multiplication of vectors with scalar.

Example Find 2A, -2A and 1/2A from the given vector A.

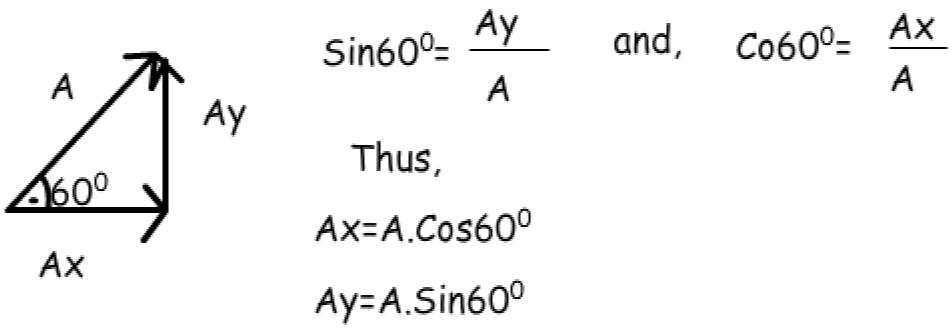



**COMPONENTS OF VECTORS**

Vectors are not given all the time in the four directions. For doing calculation more simple sometimes we need to show vectors as in the X, -X and Y, -Y components.



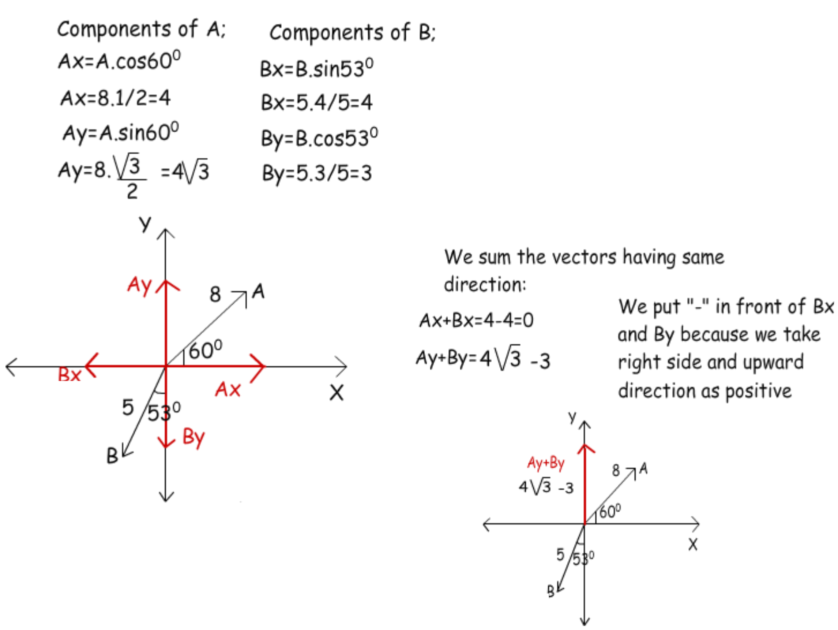

For example, look at the vector given below, it is in northeast direction. In the figure, we see the X and Y component of this vector. In other words, addition of Ax and Ay gives us A vector. We benefit from trigonometry at this point. I will give two simple equations which you can use and find the components of any given vector.



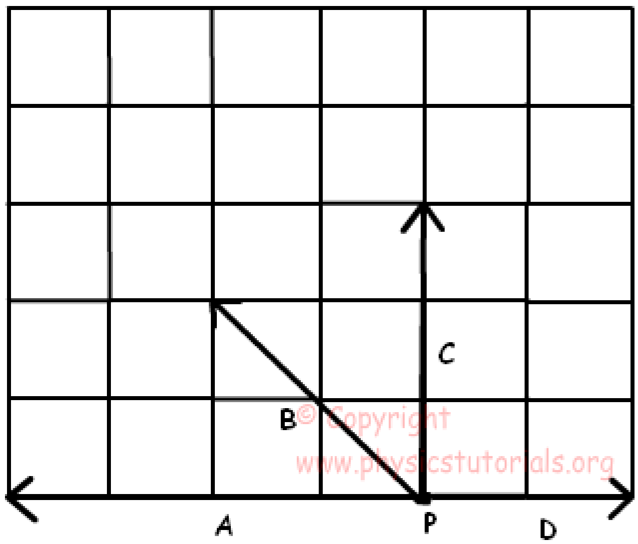

All vectors can be divided into their components. Now we solve an example and see how we use this technique.

Example Find the resultant vector of A and B given in the graph below. ($\sin 30^{\circ}=1/2$, $\sin 60^{\circ}=\sqrt{3}/2$, $\sin 53^{\circ}=4/5$, $\cos 53^{\circ}=3/5$)

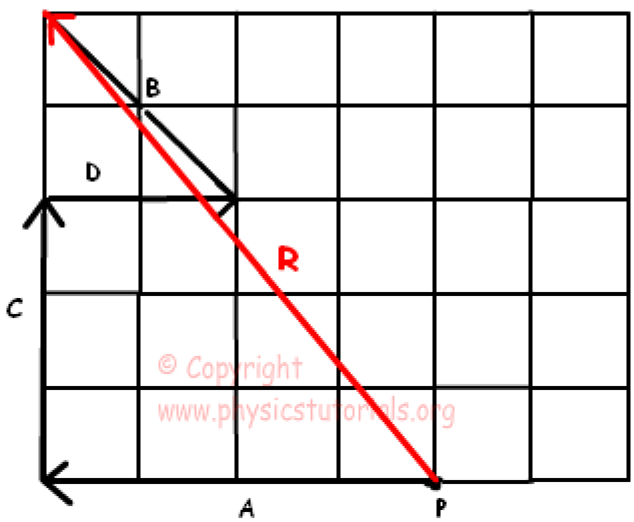
We use trigonometric equations first and find the components of the vectors then, make addition and subtraction between the vectors sharing same direction.



**Example:** Find resultant of the following forces acting on an object at point P in figure given below.




We add all vectros to find resultant force. Start with vector A and add vector C to it. After that, add vector D and C and draw resultant vector by the starting point to the end. Examine given solution below, resultant force is given in red color.

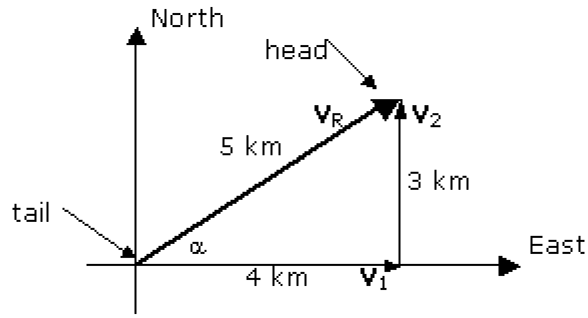



**1.- VECTORS ADDITION. GRAPHIC METHOD.**

To **add scalars** like mass or time, ordinary arithmetic is used.

If two **vectors** are in the **same line** we can also use arithmetic, but**not** if they are not in the same line. Assume for example you walk 4 km to the East and then 3 km to the

North, the resultant or net displacement respect to the start point will have a magnitude of 5 km and an angle $\alpha = 36.87^o$ with the positive x direction. See figure.




The resultant displacement $\mathbf{V}_R$ , is the sum of vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, that is we write

$\mathbf{V}_R = \mathbf{V}_1 + \mathbf{V}_2$  This is a vector equation.

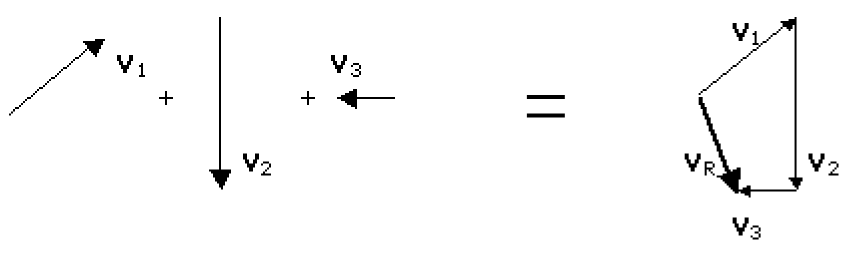
The general rule to **sum vectors** in a **graphic** way (**geometrically**) which is in fact the **definition** how vectors are added, is the following:

(1) Use a same scale for the magnitudes.
(2) Draw one of the vectors, say $\mathbf{V}_1$.
(3) Draw the other vector $\mathbf{V}_2$, placing its tail on the head of the first one, making sure to keep its direction.
(4) The sum or resultant of the vectors is the arrow drawn from the tail of the first vector to the head of the second vector.

This method is called **vector addition** from **tail to head**.

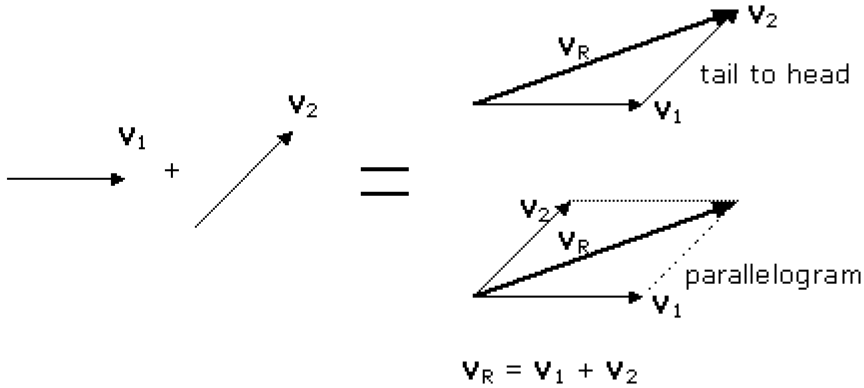
Notice that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$, that is, the order does not matter.

This **tail to head method** can be extended to three or more vectors. Suppose we want to add the vectors $\mathbf{V}_1$, $\mathbf{V}_2$ and $\mathbf{V}_3$ shown below:



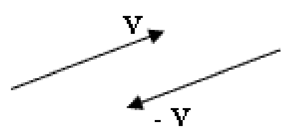

$\mathbf{V}_R = \mathbf{V}_1 + \mathbf{V}_3 + \mathbf{V}_3$ is the resultant vector outlined with a heavy line.

A second method to **add two vectors** is the **parallelogram rule** equivalent to the tail to head method. In using this parallelogram rule the two vectors are drawn from a common origin and a parallelogram is formed using the two vectors as adjacent sides. The resultant is the diagonal drawn from the common origin.




**2.- SUBTRACTION OF VECTORS**

Given a vector **V** it is defined the negative of this vector (-**V**) as a vector with the same magnitude as **V** but opposite direction:




The difference of two vectors **A** and **B** is defined as per this equation:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

So we can use the addition rules to subtract vectors.

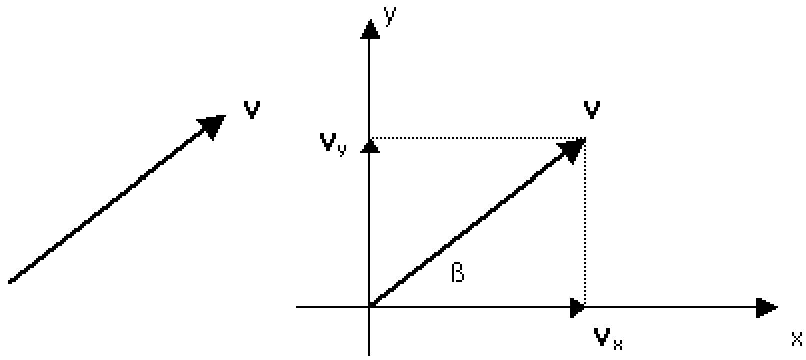
**3.- MULTIPLICATION OF A VECTOR BY A REAL NUMBER.**

A vector **V** can be multiplied by a real number c. This product is defined in such a way that c**V** has the same direction as **V** and magnitude cV. If c is positive, the sense is not altered. If c is negative, the sense is exactly opposite to **V**.

**ANALYTIC METHOD, VECTORS ADDITION.**

**1.- COMPONENTS**

The graphical sum often has not enough exactitude and is not useful when the **vectors** are in **three dimensions**. As every vector can be represented as the sum of two other vectors, these vectors are called the **components** of the original vector. Usually the components are chosen along two mutually perpendicular directions. For example, assume the vector **V** below in the figure. It can be split in the component $\mathbf{V}_x$ parallel to the x axis and the component $V_y$ parallel to the y axis.




We use coordinate axis x-y with origin at the tail of vector **V**. Notice that $\mathbf{V} = \mathbf{V}_x + \mathbf{V}_y$ according to the parallelogram rule.

The magnitudes of $\mathbf{V}_x$ and $\mathbf{V}_y$ are denoted $V_x$ and $V_y$, and are numbers, positive or negatives as they point at the positive or negative side of the x-y axis.

Notice also that $V_x = V\cos\beta$ and $V_y = V\mathrm{sen}\,\beta$.

**2.- UNIT VECTORS**

Vector quantities can often be expressed in terms of unit vectors. A unit vector is a vector whose magnitude is equal to one and dimensionless. They are used to specify a determinated direction. The symbols **i**, **j** y **k** represent unit vectors pointing in the directions x, y and z positives, respectively.

Now **V** can be written $\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j}$.

If we need to add the vector $\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j}$ with the vector $\mathbf{B} = B_x\,i + B_y\,\mathbf{j}$ we write

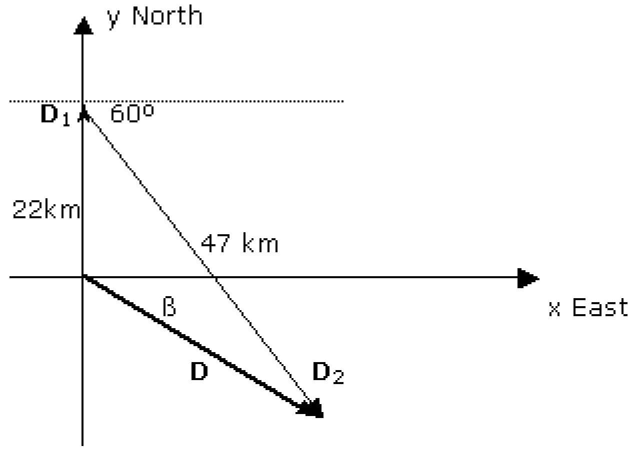
$\mathbf{R} = \mathbf{A} + \mathbf{B} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + B_x\,\mathbf{i} + B_y\,\mathbf{j} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j}$.

The components of **R** are $R_x = A_x + B_x$ and $R_y = A_y + B_y$

**Exercise, Example:** Use of **components** and **unit vectors**.

A boyscout walks 22 km in North direction, and then he walks in direction 60º Southeast during 47.0 km. Find the components of the resulting vector displacement from the starting point, its magnitude and angle with the x axis.

**Solution:** The two displacements are shown in the figure, where we choose the positive x axis pointing to East and the positive y axis pointing to North.




The resultant displacement **D** is the sum of $\mathbf{D}_1$ and $\mathbf{D}_2$.

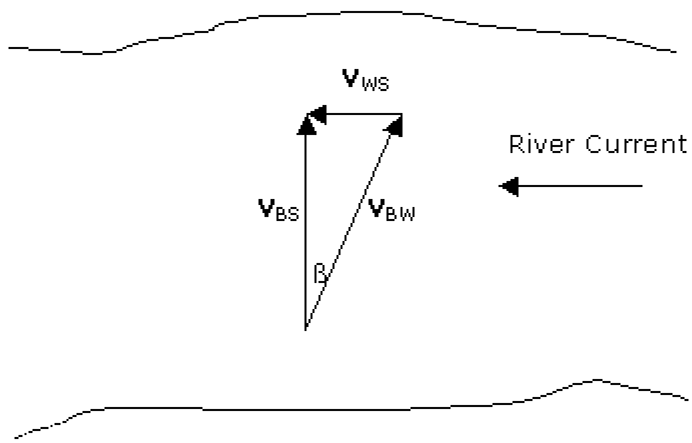
Using unit vectors:

$\mathbf{D}_1 = 22\ \mathbf{j}$
$\mathbf{D}_2 = 47\cos 60^\circ\ i - 47\text{sen}60^\circ\ \mathbf{j}$
Then $\mathbf{D} = \mathbf{D}_1 + \mathbf{D}_2 = 22\ \mathbf{j} + 47\cos 60^\circ\ \mathbf{i} - 47\text{sen}60^\circ\ \mathbf{j} = 23.5\ \mathbf{i} - 18.7\ \mathbf{j}$
and the resultant vector is completely specified with an x component $D_x = 23.5$ km and a y component $D_y = -18.7$ km. (Note $D_x$ and $D_y$ are scalars).

The same resultant vector can be specified giving its magnitude and angle:
$D^2 = D_x^2 + D_y^2 = (23.5\text{ km})^2 + (-18.7\text{ km})^2$ finding D = 30 km.
$\tan \beta = D_y/D_x = -18.7/23.5 = -0.796$ finding $\beta = -38.5^\circ$ (under the x axis) or 38.5 Southeast.

**Application Problems: Use of Addition Vector Tools to Solve Relative Velocity**

**Vector Problems, Example One.-**
A motorboat velocity is 20 km/h in still water. If the boat must travel straight to the nearest shore in a river whose current is 12 km/h, ¿What up stream angle must the bow boat point at?

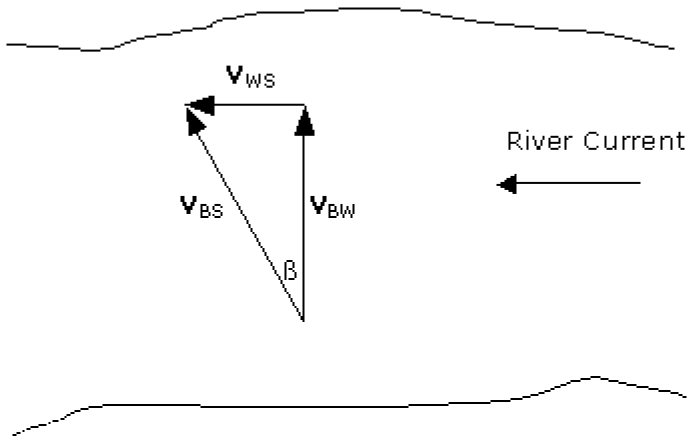



Before attempting to solve this problem, it is useful to do some considerations:
- Whenever a velocity is mentioned, it is necessary to specify what is its frame of reference to measure it. This is a case where we have relative velocities and the tool to find the resultant or the components is the vector sum.
- It is helpful to use an identification procedure that uses two sub indexes: the first sub index refers to the object and the second one to the frame of reference in which that velocity is measured. In this example $\mathbf{V}_{BW}$ is the velocity of the **B**oat relative to the **W**ater, $\mathbf{V}_{BS}$ is the Boat velocity relative to the **S**hore and $\mathbf{V}_{WS}$ is the Water velocity relative to the Shore. Notice $\mathbf{V}_{BW}$ is produced by the boat motor, instead $\mathbf{V}_{BS}$ is $\mathbf{V}_{BW}$ plus the current effect. Hence, the boat velocity relative to the shore $V_{BS}$, is

$$\text{(A) } \mathbf{V}_{BS} = \mathbf{V}_{BW} + \mathbf{V}_{WS}$$

The sentence "a motorboat velocity is 20 km/h in still water" means $V_{BW}$ = 20 km/h, and the sentence "a river whose current is 12 km/h" means $V_{WS}$ = 12 km/h. Notice $\mathbf{V}_{BS}$ points directly straight to the opposite shore as wanted. The angle $\beta$ can be obtained from the rectangular triangle in the figure:
sen $\beta = V_{WS}/V_{BW} = 12/20 = 0.6$ then $\beta = 36.87^o$. The bow boat must point at an angle of $36.87^o$ up stream in order to cross the river directly to the other shore.

**Vector Problems, Example 2.-**
A boat velocity is 2 m/s in still water. a) If the boat points the bow straight to the opposite shore to cross the river whose current is 1 m/s, what is the velocity, in magnitude and direction, of the boat relative to the shore? b) What is the boat position relative to its starting point, after 3 min?




a) The boat velocity relative to the shore $\mathbf{V}_{BS}$, is the sum of its velocity relative to the water $\mathbf{V}_{BW}$, and the water velocity relative to the shore $\mathbf{V}_{WS}$ :

$$\mathbf{V}_{BS} = \mathbf{V}_{BW} + \mathbf{V}_{WS}$$

As $\mathbf{V}_{BW}$ and $\mathbf{V}_{WS}$ are the sides of a rectangular triangle,
$V_{BS}^2 = V_{BW}^2 + V_{WS}^2$ or
$V_{BS}^2 = (2\text{m/s})^2 + (1\text{m/s})^2$
i.e. $V_{BS} = 2.24$ m/s.
Also, $\tan\beta = V_{WS}/V_{BW} = 1/2$ ,
then $\beta = 26.57^o$.
b) To calculate the position after 3 min we can use
$\mathbf{D} = \mathbf{V}t$,
with **D** the vector displacement, **V** the vector velocity and t the time, an scalar. The direction of **D** is the same as **V** and its magnitude is
$V\cdot t = 2.24\ \text{m/s}\cdot 3\cdot 60\ \text{s}$
$= 403.2$ m.
The boat position is then at 403.2 m from the starting point and at a direction $26.57^o$ down stream transverse to it

**Scalar quantities:** **پرسیار و وەلام**

- mass
- length
- time
- speed
- temperature
- electric current

**Vector quantities:**

- force
- velocity
- acceleration
- displacement
- magnetic induction

**Question**

Define Momentum. It is a scalar and vector quantity ? give its unit and dimension

**Solution**

Momentum of a body can be defined as the product of mass of the body and velocity of the body. It means momentum of a body is directly proportional to the mass of the body. Momentum of a body is also directly proportional to the Velocity of the body. The SI unit of momentum is kilogram metre per second.

It is a vector quantity.

**Problem statement:**

Given the vectors: **A** = 3**i** + 2**j** – **k** and **B** = 5**i** +5**j**.

Determine:

a. Their magnitude.
b. The direction of **B**.
c. **A** + **B**
d. **A** -2 **B**
e. A unit vector parallel to **A**.

f. A vector of magnitude 2 and opposite to **B**

Solution:

The magnitude of **A** is given by:

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$|\vec{A}| = \sqrt{3^2 + 2^2 + (-1)^2} = 3.74$$

Similarly, the magnitude of **B** is:

$$|\vec{B}| = B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$|\vec{B}| = \sqrt{5^2 + 5^2 + 0^2} = 7.07$$

**The magnitude of a vector is always a positive number**.

Example:

$$\vec{A} - 2\vec{B} = (A_x - 2B_x)\vec{i} + (A_y - 2B_y)\vec{j} + (A_z - 2B_z)\vec{k}$$

$$\vec{A} - 2\vec{B} = (3 - 2\cdot 5)\vec{i} + (2 - 2\cdot 5)\vec{j} + (-1 - 2\cdot 0)\vec{k}$$

$$\vec{A} - 2\vec{B} = -7\vec{i} + 8\vec{j} - \vec{k}$$

**Example :**

A ball is thrown with an initial velocity of 70 cm per second., at an angle of 35°35° with the horizontal. Find the vertical and horizontal components of the velocity.

Let v represent the velocity and use the given information to write v in unit vector form:

v =70(cos(35°))i+70(sin(35°))j

=70(cos(35°))i+70(sin(35°))j

Simplify the scalars, we get:

v ≈57.34i+40.15j

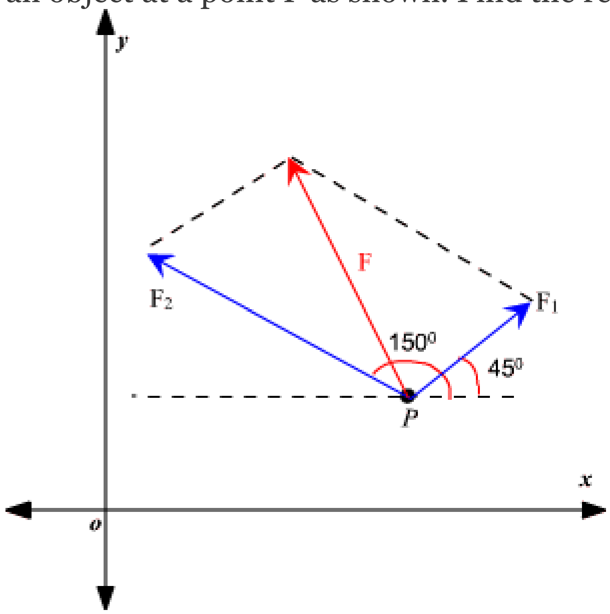
≈57.34i+40.15j

Since the scalars are the horizontal and vertical components of v ,

Therefore, the horizontal component is 57.34 cm per second and the vertical component is 40.15 cm per second.

**Example :**

Two forces F1 and F2 with magnitudes 20 and 30 Newtons , respectively, act on an object at a point P as shown. Find the resultant forces acting at P .




First we write F1 and F2 in component form:
v≈57.34i+40.15j≈57.34i+40.15j
Simplify the scalars, we get:

F1=(20cos(45°))i+(20sin(45°))j
=20(2√2)i+20(2√2)j
=102√i+102√j
F2=(30cos(150°))i+(30sin(150°))j
=30(−3√2)i+30(12)j
=−153√i+15j
So, the resultant force F is
F=F1+F2=(102√ i+102√j)+(−153√ i+15j)
=(102√−153√)i+(102√+15)j
≈−12i+29j

# Chapter Two: Newton's Laws of Motion

**Newton's laws of motion**: Three statements describing the relations between the forces acting on a body and the motion of the body, first formulated by English physicist and mathematician Isaac Newton, which are the foundation of classical mechanics.

In his *Principia*, Newton reduced the basic principles of mechanics to three laws:

1. Newton's first law states that if a body is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force.
2. **Newton's second law:**

   It states that the time rate of change of the momentum of a body is equal in both magnitude and direction to the force imposed on it. The momentum of a body is equal to the product of its mass and its velocity.

   Momentum, like velocity, is a vector quantity, having both magnitude and direction. A force applied to a body can change the magnitude of the momentum or its direction or both.

   Newton's second law is one of the most important in all of physics.

   For a body whose mass $m$ is constant, it can be written in the form

   $\boldsymbol{F = ma}$,

   where $F$ (force) and $a$ (acceleration) are both vector quantities.
3. To every action there is always opposed an equal reaction. **Newton's third law: The law of action and reaction.** Newton's third law states that when two bodies interact, they apply forces to one another that are equal in magnitude and opposite in direction. The third law is also known as the law of action and reaction. For example, a book resting on a table applies a downward force equal to its weight on the table. According to the third law, the table applies an equal and opposite force to the book. This force occurs because the weight of the book causes the table to deform slightly so that it pushes back on the book like a coiled spring.

**Newton's law of gravitation**:

It states that any particle of matter in the universe attracts any other with a force varying directly as the product of the masses and inversely as the square of the distance between them.

In symbols, the magnitude of the attractive force $F$ is equal to $G$ (the gravitational constant($G=6.67\times10^{-11}$ $N.m^2/kg^2$), multiplied by the product of the masses ($m_1$ and $m_2$) and divided by the square of the distance $R$:

$F = G(m_1m_2)/R^2$.

Examples:

Ex.1: A force of 20 newtons acted upon a body of mass 8 Kg place on a horizontal surface. Calculate the acceleration.

Ex2: A force of 35 newtons acted on a body on a horizontal surface. The body moved with a constant acceleration of $3.5 m/sec^2$. Calculate the mass of the body.

Ex.3: Two forces 5newtons and 3 newtons acted on a body of mas 2 Kg put on a horizontal surface. The two forces acted in opposite directions. Calculate the acceleration.

Ex. 4: two bodies of masses $m_1$=100Kg and $m_2$= 200kg . the distance between the center of the two bodies was 40m. Calculate the gravitational force between the two bodies.
Ex.5: calculate the gravitational force between the earth and the moon?
Ex.6: calculate the gravitational force between the electron and proton of Hydrogen atom.
**Ex.7:** What is the gravitational force between the Sun and the Earth?
The mass of the Sun = 2x $10^{30}$ Kg.
mass of the Earth = 6 x $10^{24}$ Kg
G = 6.67 x $10^{-11}$ $N.m^2/kg^2$
The distance between the Sun and the Earth=1.5 x $10^{11}$ m.

## What are some daily life examples of Newton's second law of motion?

Newton's second law of motion explains how force can change the acceleration of the object and how the acceleration and mass of the same object are related. Therefore, in daily life, if there is any change in the object's acceleration due to the applied force, they are examples of Newton's second law.

- Acceleration of the rocket is due to the force applied known as thrust and is an example of Newton's second law of motion.
- Another example of Newton's second law is when an object falls from a certain height, the acceleration increases because of the gravitational force.

## Chapter Three: Equations of Motion----Mechanics

## Motion in a straight line with constant acceleration

Suppose a particle moving in a straight line with a constant or uniform acceleration( a )for a time interval t.
During this time its velocity will change from u to v.
The particle travels a distance s.
If you know any three of the quantities (a, t, u, v ,s) then you can calculate the remaining two.

| | |
|---|---|
| The initial speed/velocity is | u. |
| The final speed/velocity is | v. |
| The acceleration is | a. |
| The distance/displacement is | s. |
| The time take taken is | t. |

The equations:

$v = u + at$. (no s) (1)

$s = ut + ½ at^2$. (no v) (2)

$s = vt – ½ at^2$. (no u) (3)

$s = ½ ( u + v ) t$. (no a) (4)

$v^2 = u^2 +2as$. (no t) (5)

**Example 1** ( units: metres and seconds )

(i) Find v when u = 10, a = 6 and t = 2.
(ii) Find s when u = 10, a = 8 and t = 2.
(iii) Find s when v = 30, a = 4 and t = 5.
(iv) Find s when u = 10, v = 8 and t = 2.
(v) Find v when u = 10, a = 5 and s = 6.

**Solutions**

(i) $v = u + at$ ; $v = 10 + 6 \times 2 = 10 + 12 = 22\ ms^{-1}$
(ii) $s = ut + ½ at^2$ ; $s = 10 \times 2 + 0.5 \times 8 \times 2^2 = 20 + 16 = 36$ m.
(iii) $s = vt – ½ at^2$ ; $s = 30 \times 5 - 0.5 \times 4 \times 5^2 = 150 – 50 = 100$ m.
(iv) $s = ½ ( u + v ) t$ ; $s= ½ ( 10 + 8 ) \times 2 = 18$ m.
(v) $v^2 = u^2 +2as$ ; $v^2 = 10^2 + 2 \times 5 \times 6 = 100 + 60 =160$
$v = (160)^{1/2} = 12.6\ ms^{-1}$.

**Example 2**

Decide which equations to use in each of these situations.

| | |
|---|---|
| (i) Given u, s, a ; find v. | (ii) Given u, t, a ; find v. |
| (iii) Given u, t, a ; find s. | (iv) Given u, s, v ; find t. |
| (v) Given u, s, v ; find a. | (vi) Given u, s, t ; find a. |
| (vii) Given u, a, v ; find s. | (viii) Given a, s, t ; find v. |

**Solutions**

| | |
|---|---|
| (i) $v^2 = u^2 +2as$ | (ii) $v = u + at$ |
| (iii) $s = ut + ½ at^2$ | (iv) $s = ½ ( u + v ) t$ |
| (v) $v^2 = u^2 +2as$ | (vi) $s = ut + ½ at^2$ |
| (vii) $v^2 = u^2 +2as$ | (viii) $s = vt – ½ at^2$ |

***Example 3***

A particle is moving in a straight line from O to A with a constant acceleration of 2 ms $^{-2}$. Its velocity at A is 30 ms$^{-1}$ and it takes 15 seconds to travel from O to A. Find

(a) Particle's velocity at O,

(b) Distance OA.

***Solution***

**(a) Given a, v, t; needed u: Equation: $v = u + at$**

$30 = u + 2 \times 15$ then $u = 0$

The velocity at O is 0 ms $^{-1}$

(b) Known a, u, v, t; needed s: equations: $s = ut + ½ at^2$ or

$s = ½ ( u + v ) x t$

$s = 0 \times 15 + 0.5 \times 2 \times 15^2 = 225$

The distance OA is 225 m.

Example 4

A boy on a skateboard is travelling up a hill. He experienced a constant deceleration of magnitude 2 ms $^{-2}$ . Given that his speed at the bottom of the hill was 10 ms$^{-1}$, determine how far he will travel before he comes to rest.

Solution

Known quantities are a, u, v; needed s:

Required equation $v^2 = u^2 +2as$

$0 = 10^2 + 2 \times -2 \times s = 100-4s$ then $s = 100/4 = 25$

The boy travels a distance of 25 m before coming to rest.

## <u>Exercises</u>

فیزیا - قۆناغی یەکەم- پرسیارەکانی بەشی سێیەم

The equations:

$v = u + at.$ (1)

$s = ut + ½ at^2$ (2)

$s = vt – ½ at^2$ (3)

$s = ½ ( u + v ) x t$ (4)

$v^2 = u^2 +2as$ (5)

1 A particle moves in a straight line with uniform acceleration 5 ms $^{-2}$ . It starts from rest when t = 0. Find its velocity when t = 3 s.

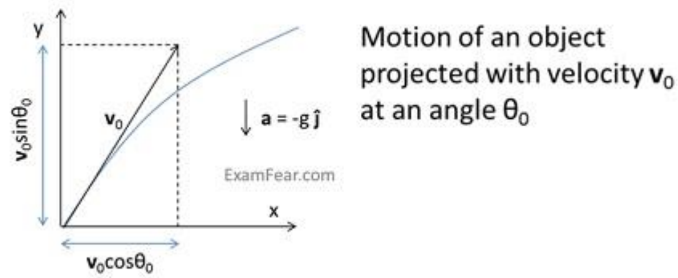
| | |
|---|---|
| 2 | A particle moves in straight line. When t = 0 its velocity is 3 $ms^{-1}$ . When t = 4 its velocity is 12 $ms^{-1}$ . Find its acceleration. |
| 3 | A particle moves in straight line with constant retardation 4 $ms^{-2}$ . When t = 3 s its velocity is 5 $ms^{-1}$ . Find its initial velocity. |
| 4 | A particle moves in a straight line with uniform acceleration 5 $ms^{-2}$ . How long will it take for the particle's velocity to increase from 2 $ms^{-1}$ to 24 $ms^{-1}$ ? |
| 5 | A particle starts from rest and moves in a straight line with constant acceleration 4 $ms^{-2}$ for 3 s. How far does it travel ? |
| 6 | A particle moves along a line PQ with constant acceleration 3 $ms^{-2}$ . If PQ is 2 m and the particle takes 0.5 s to travel from P to Q , what was its velocity at P? |
| 7 | A particle moves in a straight line with uniform acceleration 8 $ms^{-2}$ . If its initial velocity is 2 $ms^{-1}$ how long will it take to travel 6 m ? |
| 8 | A particle moving in a straight line experience a constant retardation of 6 $ms^{-2}$ . How long will it take to decrease its speed from 20 $ms^{-1}$ to 8 $ms^{-1}$ and how far will it travel while doing so ? |
| 9 | A particle is moving with uniform acceleration. If it starts from rest and 5 s later has a speed of 18 $ms^{-1}$ find the distance it has travelled. |
| 10 | A particle is moving in a straight line with uniform acceleration . If it travels 120 m while increasing speed from 5 $ms^{-1}$ to 25 $ms^{-1}$ find its acceleration. |
| 11 | A car is accelerating uniformly while travelling along a straight road. Its speed increases from 8 $ms^{-1}$ to 22 $ms^{-1}$ in 10 s. Find the distance travelled during this time and the acceleration of the car. |

**Chapter Four: Projectile Motion**

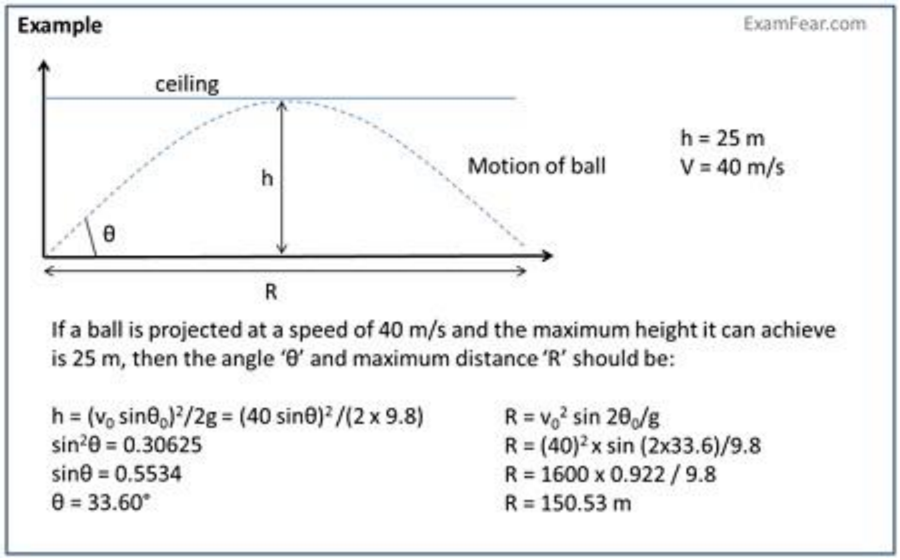
An object that becomes airborne after it is thrown or projected is called **projectile**. Example, football..


Projectile Motion

- Projectile motion consist of two parts.
  1- Horizontal motion of no acceleration;
  2- 2- Vertical motion of constant acceleration due to gravity.
- Projectile motion is in the form of a parabola, $y = ax + bx^2$.




| **Quantity** | **Value** |
|---|---|
| Components of velocity at time t | $v_x = v_0 \cos\theta_0$<br>$v_y = v_0 \sin\theta_0 – gt$ |
| Position at time t | $x = (v_0 \cos\theta_0)t$<br>$y = (v_0 \sin\theta_0)t – ½ gt^2$ |
| Equation of path of projectile motion | $y = (\tan \theta_0)x – gx^2/2(v_0 \cos\theta_0)^2$ |
| Time of maximum height | $t_m = v_0 \sin\theta_0 /g$ |
| Time of flight | $2\ t_m = 2\ (v_0 \sin\theta_0 /g)$ |
| Maximum height of projectile | $h_m = (v_0 \sin\theta_0)^2/2g$ |
| Horizontal range of projectile | $R = v_0^2 \sin 2\theta_0/g$ |
| Maximum horizontal range ($\theta_0$=45°) | $R_m = v_0^2/g$ |




**Problem 1**

An object is launched at a velocity of 20 m/s in a direction making an angle of 25° upward with the horizontal.

a) What is the maximum height reached by the object?

b) What is the total flight time (between launch and touching the ground) of the object?

c) What is the horizontal range (maximum x above ground) of the object?

d) What is the magnitude of the velocity of the object just before it hits the ground?

<u>Solution to Problem 1:</u>

a) The formulas for the components $V_x$ and $V_y$ of the velocity and components x and y of the displacement are given by :

$V_x = V_0 \cos(\theta)$ $\quad V_y = V_0 \sin(\theta) - g\,t$

$x = V_0 \cos(\theta)\,t$ $\quad y = V_0 \sin(\theta)\,t - (1/2)\,g\,t^2$

In the problem $V_0 = 20$ m/s, $\theta = 25°$ and $g = 9.8$ m/s$^2$.

The height of the projectile is given by the component y, and it reaches its maximum value when the component $V_y$ is equal to zero.

$V_y = V_0 \sin(\theta) - g\,t = 0$

solve for t

$t = V_0 \sin(\theta) / g = 20 \sin(25°) / 9.8 = 0.86$ seconds

Maximum height is:

$y = 20 \sin(25°)(0.86) - (1/2)(9.8)(0.86)^2 = 3.64$ meters

b) The time of flight is:

$V_0 \sin(\theta)\,t - (1/2)\,g\,t^2 = 0$

Solve for t

$t(V_0 \sin(\theta) - (1/2)\,g\,t) = 0$

either $t = 0$ or $t = 2 V_0 \sin(\theta) / g$

Time of flight $= 2 (20) \sin(\theta) / g = 1.72$ seconds.

c) The time of flight $t = 2 V_0 \sin(\theta) / g$. The horizontal range is the horizontal distance given by x at t .

Range $= x = V_0 \cos(\theta)\,t = 2 V_0 \cos(\theta) V_0 \sin(\theta) / g = V_0^2 \sin(2\theta) / g = 20^2 \sin(2x25°) / 9.8 = 31.26$ meters

d) The object hits the ground at t =2 V0 sin(θ) / g
The components of the velocity at t are given by
$V_x = V_0 \cos(\theta)$ $V_y = V_0 \sin(\theta) - g\ t$
The components of the velocity at t = 2 V0 sin(θ) / g are given by
$V_x = V_0 \cos(\theta) = 20 \cos(25°)$ $V_y = V_0 \sin(25°) - g\ (2\ V_0 \sin(25°) / g) = -V_0 \sin(25°)$
The magnitude V of the velocity is given by
$V = \sqrt{[\ V_x^2 + V_y^2\ ]} = \sqrt{[\ (20 \cos(25°))^2 + (-\ V_0 \sin(25°))^2\ ]} = V_0 = 20$ m/s

**Problem 2**

A ball is kicked at an angle of 35° with the ground.
a) What should be the initial velocity of the ball so that it hits a target that is 30 meters away at a height of 1.8 meters?
b) What is the time for the ball to reach the target?
<u>Solution to Problem 2:</u>
a)
$x = V_0 \cos(35°)\ t$
$30 = V_0 \cos(35°)\ t$
$t = 30 / V_0 \cos(35°)$
$1.8 = -(1/2)\ 9.8\ (30 / V_0 \cos(35°))^2 + V_0 \sin(35°)(30 / V_0 \cos(35°))$
$V_0 \cos(35°) = 30 \sqrt{[\ 9.8 / 2(30 \tan(35°)-1.8)\ ]}$
$V_0 = 18.3$ m/s
b)
$t = x / V_0 \cos(35°) = 2.0$ s

**Problem 3**

A ball kicked from ground level at an initial velocity of 60 m/s and an angle θ with ground reaches a horizontal distance of 200 meters.
a) What is the size of angle θ?
b) What is time of flight of the ball?

<u>Solution to Problem 3:</u>
a)
Let T be the time of flight. Two ways to find the time of flight
1) $T = 200 / V_0 \cos(\theta)$ (range divided by the horizontal component of the velocity)
2) $T = 2\ V_0 \sin(\theta) / g$
equate the two expressions
$200 / V_0 \cos(\theta) = 2\ V_0 \sin(\theta) / g$
which gives
$2\ V_0^2 \cos(\theta)\sin(\theta) = 200\ g$
$V_0^2 \sin(2\theta) = 200\ g$
$\sin(2\theta) = 200\ g / V_0^2 = 200\ (9.8) / 60^2$
Solve for θ to obtain

$\theta = 16.5°$
b)Time of flight = 200 / $V_0$ cos(16.5°) = 3.48 s

**Problem 4**
A projectile starting from ground hits a target on the ground located at a distance of 1000 meters after 40 seconds.
a) What is the size of the angle θ?
b) At what initial velocity was the projectile launched?
Solution to Problem 4:
a) Vx = $V_0$ cos (θ) = 1000 / 40 = 25 m/s
Time of flight = 2 $V_0$ sin (θ) / g = 40 s
$V_0$ sin (θ) = 20 g
Combine the above equation with the equation $V_0$ cos (θ) = 25 m/s
tan (θ) = 20 g / 25
Use calculator to find θ = 82.7 °
b)We now use any of the two equations above to find $V_0$.
$V_0$ cos (θ) = 25 m/s
$V_0$ = 25 /cos (82.7 °) = 196.8 m/s
**Problem 5**
The trajectory of a projectile launched from ground is given by the equation y = -0.025 $x^2$ + 0.5 x, where x and y are the coordinate of the projectile on a rectangular system of axes.
Find the initial velocity and the angle at which the projectile is launched.
Solution to Problem 5:
$y = \tan(\theta) x - (1/2) (g / (V_0 \cos (\theta))^2 ) x^2$
hence tan(θ) = 0.5 which gives θ = arctan(0.5) = 26.5 °
$-0.025 = -0.5 (9.8 / (V_0 \cos (26.5°))^2 )$
Solve for $V_0$ to obtain $V_0$ = 15.6 m/s
**Problem 6**
Two balls A and B of masses 100 grams and 300 grams respectively are pushed horizontally from a table of height 3 meters. Ball has is pushed so that its initial velocity is 10 m/s and ball B is pushed so that its initial velocity is 15 m/s.
a) Find the time it takes each ball to hit the ground.
b) What is the difference in the distance between the points of impact of the two balls on the ground?
Solution to Problem 6:
a) The two balls are subject to the same gravitational acceleration and therefor will hit the ground at the same time t found by solving the equation
$-3 = -(1/2) g t^2$
t = √ (3(2)/9.8) = 0.78 s
b) Horizontal distance XA of ball A
XA = 10 m/s * 0.78 s = 7.8 m
Horizontal distance XB of ball B
XB = 15 m/s * 0.78 s = 11.7 m
Difference in distance XA and XB is given by
|XB - XA| = |11.7 - 7.8| = 3.9 m

## Chapter Five: Linear Momentum

The linear momentum of an object is defined as the product of its mass and its velocity. It is measured in units of Kg.m/sec. The relation for linear momentum is:

$$\vec{P} = m\vec{v}$$

Where P and v are vector quantities. So an object will have have large momentum due to large mass, large velocity, or both.

Change object's Momentum:

The momentum of an object changes it its mass or velocity changes, or both. So, we can obtain a relation for the amount of change by re-writing newton's second law, as follow:

$$(\vec{F}_{\text{Net}} = \text{m x } \vec{a})$$

$$\vec{F}_{\text{Net}} = \text{m}\vec{a} \longrightarrow \vec{F}_{\text{Net}} = \frac{\Delta(m\vec{v})}{\Delta t}$$

Thus the general form of newton's second law says that the net force is equal to the rate of change of momentum.

In order to change the momentum of an object, a force must be applied.

If we now multiply both sides of this equation by the time interval $\boldsymbol{\Delta t}$ , we get an equation that tells us how to produce a change in momentum.

$$\vec{F}_{\text{Net}}\Delta\text{t} = \Delta(\text{m x } \vec{v})$$

This relationship tell us that the change in momentum is the net force multiplied by the certain time interval.. The change in momentum is called **Impulse**. Impulse is a vector quantity, it has the same direction as the applied force. The unit od impulse is N.s and is equivalent to the change of momentum Kg.m/sec .

Impulse is the product of two things, so there are many ways to change the momentum to the same value.
For example, if we want impulse of 10Ns., we can:
Exert a force of 5 N on the object for 2 sec. Or
Exert 100N for 0.1 sec
Each will produce the same impulse.
Example: Suppose you had to jump from a window. Would you prefer to jump onto a wooden surface or onto a concrete surface? Why?
So , in brief, the impulse is

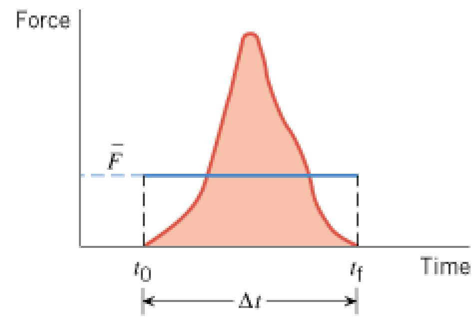
**Impulse = average force x time of contact. Or the impulse is the change in momentum P.**

**I= FΔ t = Δ P**

$$\vec{F}\Delta t = \Delta\vec{P}$$

$$\vec{F}dt = d\vec{P}$$

$$\vec{I} = \int_{t_i}^{t_f} d\vec{p} = \int_{t_1}^{t_2} \vec{F}(t)dt$$



(b)

Therefore the impulse is area and the graph.

Example 1: What is the momentum of a ball with mass 5kg and velocity 10m/s?

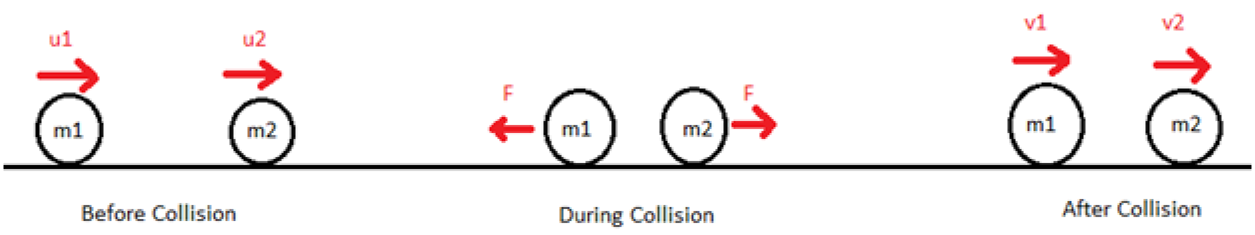
Momentum = mass x velocity
Momentum = 5Kgx 10 m/s = 50 Kg.m/s

Example 2; What will be the change in momentum caused by a net force of 120N acting on an object for 2 seconds?

Change in momentum = net force x time interval
Change in momentum= 120N x 2 s = 240 N.

**Conservation of linear momentum:**

Newton's Second law relates force with the rate of change of momentum. According to the law, force is directly proportional to the rate of change in momentum. $F \propto \Delta P$




We will use this to state the law of conservation of momentum. According to this, if the net force acting on the system is zero, then the system's momentum remains conserved.

In other words, the change in momentum of the system is zero. According to the second law, we can see as F = 0, so it will also be zero.

Let's take the following example:
We consider $m_1$ and $m_2$ as our system. So during the collision, the net force on the system is zero, and hence we can conserve the system's momentum. The equation for momentum will be:

*Initial momentum* = $m_1u_1 + m_2u_2$

*Final momentum* = $m_1v_1 + m_2v_2$

*So, according to the conservation of momentum,*

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

But one thing to take care is that conservation is only true for a system and not one body because if we consider only a single body $m_1$ or $m_2$, then the net force will be acting on it, so we cannot write

$$m_1u_1 \neq m_1v_1 \text{ or } m_2u_2 \neq m_2v_2.$$

Discuss the law of conservation of momentum. State its unit.

The law of conservation of momentum states that when two objects collide in an isolated system, the total momentum before and after the collision remains equal. This is because the momentum lost by one object is equal to the momentum gained by the other.
In other words, if no external force is acting on a system, its net momentum gets conserved.
The unit of momentum in the S.I system is kgm/s or simply Newton Second(Ns).

**Conservation of Linear Momentum :**

Example

Two bodies of mass m1 and m2 are moving in opposite directions with the velocities v1 and v2. If they collide and move together after the collision, we have to find the velocity of the system Vfinal.

Since there is no external force acting on the system of two bodies, momentum will be conserved.

Initial momentum = Final momentum

$(m1v1 – m2v2) = (m1+m2)V_{Final}$

Thus   Vfinal = (m1v1-m2v2)/(m1+m2)

From this equation, we can easily find the final velocity of the system.

Definition of conservation of momentum
For two or more bodies in an isolated system acting upon each other, their total momentum remains constant unless an external force is applied. Therefore, momentum can neither be created nor destroyed.

The principle of conservation of momentum is a direct consequence of Newton's third law of motion.

Derivation of Conservation of Momentum

Newton's third law states that for a force applied by an object A on object B, object B exerts back an equal force in magnitude, but opposite in direction.

This idea was used by Newton to derive the law of conservation of momentum.

Consider two colliding particles A and B whose masses are $m_1$ and $m_2$ with initial and final velocities as $u_1$ and $v_1$ of A and $u_2$ and $v_2$ of B.

The time of contact between two particles is given as t.

A=m1(v1−u1)   (change in momentum of particle A)

B=m2(v2−u2)   (change in momentum of particle B)

FBA=−FAB   (from third law of motion)
Then

FBA=m2∗a2=m2(v2−u2)t
and
FAB=m1∗a1=m1(v1−u1)t

Then
$m2(v2-u2)t=-m1(v1-u1)t$

Therefore
$m1u1+m2u2 = m1v1+m2v2$

Therefore, above is the equation of law of conservation of momentum where

**m1u1+m2u2** is the representation of total momentum of particles A and B before the collision and

**m1v1+m2v2** is the representation of total momentum of particles A and B after the collision.

Solved Problems on Law of Conservation of Momentum

Q1. There are cars with masses 4 kg and 10 kg respectively that are at rest. The car having the mass 10 kg moves towards the east with a velocity of 5 $m.s^{-1}$. Find the velocity of the car with mass 4 kg with respect to ground.
Ans: Given,

$m_1 = 4$ kg

$m_2 = 10$ kg

$v_1 = ?$

$v_2 = 5\ m.s^{-1}$

$P_{initial} = 0$, as the cars are at rest

$P_{final} = p_1 + p_2$

$P_{final} = m_1.v_1 + m_2.v_2$

$= (4\ kg).(v_1) + (10\ kg).(5\ m.s^{-1})$

$= 4kg \times V1 + 50\ kg.m.s^{-1}$

We know from the law of conservation of momentum that,

$P_{initial} = P_{final}$

$0=4\ kg.v_1+50\ kg.m.s^{-1}$

Then

$V1= - 50/4$

$v_1 = 12.5\ m.s^{-1}$

The negative sign means the opposite direction

Q2. Find the velocity of a bullet of mass 5 grams which is fired from a pistol of mass 1.5 kg. The recoil velocity of the pistol is 1.5 $m.s^{-1}$.
Ans: Given,

Mass of bullet, $m_1$ = 5 gram = 0.005 kg

Mass of pistol, $m_2$ = 1.5 kg

The velocity of a bullet, $v_1$ = ?

Recoil velocity of pistol, $v_2$ = 1.5 $m.s^{-1}$

Using law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Here, Initial velocity of the bullet, $u_1 = 0$

Initial recoil velocity of a pistol, $u_2 = 0$

$\therefore$ (0.005 kg)(0) + (1.5 kg)(0) = (0.005 kg)($v_1$) + (1.5 kg)(1.5 $m.s^{-1}$)

0 = (0.005 kg)($v_1$)+(2.25 $kg.m.s^{-1}$)

Then

V1= - 2.25/0.005

$v_1$= -450 $m.s^{-1}$

Hence, the recoil velocity of the pistol is 450 $m.s^{-1}$.

The negative sign means the recoil velocity is in the opposite direction.

***Erbil – September 2022.***